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Working Paper No. 11, 2026

Durham Business School working papers series ISSN 1749-3641

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Top Trading Cycles vs. the Crawler: The Role of Obvious Strategy-Proofness*

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August 7, 2026

Abstract

Obvious strategy-proofness makes truth-telling, in theory, an obvious choice. We experimentally study its role in the object-reallocation problem with single-peaked preferences. We compare two efficient and strategy-proof mechanisms, Top Trading Cycles and the Crawler, under static and gradual implementations. In our experiment, all four mechanisms generate the same outcome under truth-telling, but only the Gradual Crawler is obviously strategy-proof. The Gradual Crawler achieves significantly higher empirical efficiency than the other mechanisms. By contrast, the Gradual TTC does not outperform the Static TTC, suggesting that gradual implementation alone is not sufficient and that the Gradual Crawler benefits from its stop-or-continue procedure.

Key words: market design; object reallocation; top trading cycles; the Crawler; obvious strategy-proofness; simplicity; laboratory experiment

JEL classification: C78, C91, D47

*We thank Yan Chen and the participants at the 4th Durham–Newcastle Economics Workshop and the 12th Annual UEA Behavioural Game Theory Workshop for their helpful comments. The experiment reported in this paper was approved by the Institutional Review Board of the Institute of Social and Economic Research (ISER), the University of Osaka (No. 20251101), and was pre-registered at AsPredicted (#269225). We gratefully acknowledge financial support from the Japan Society for the Promotion of Science (JSPS) through Grant-in-Aid for Scientific Research (KAKENHI, grant numbers 25H00388 and 25K24681) as well as support from the Joint Usage/Research Center at ISER, the University of Osaka. Tamura acknowledges financial support from LabEx Ecodec (ANR-11-LABX-0047). Authors have used Anthropic's Claude for proof-reading the manuscript.

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1 Introduction

Strategy-proofness plays a central role in mechanism design because allocation procedures are evaluated using agents' true preferences. Because those preferences are private information, the designer must rely on agents to report them. A strategy-proof mechanism removes the incentive to misreport, allowing the designer to treat reports as a reliable basis for choosing an allocation. This instrumental role is important, but its behavioral content is more delicate. A mechanism can make truthful reporting optimal without making that optimality easy to recognize. Agents may misunderstand the incentives, find the reporting task difficult, or use simple heuristics that lead them away from truthful behavior. A mechanism that performs well under truthful reporting may therefore perform less well in practice.

This paper asks whether the implementation of a strategy-proof rule affects behavior and welfare, holding fixed the rule and the allocation it selects under truthful reporting. The question is motivated by obvious strategy-proofness (Li, 2017), which strengthens strategy-proofness by requiring the optimality of truthful behavior to be transparent. The concept suggests that incentive compatibility may depend not only on the allocation rule but on how agents interact with it.

The canonical obviously strategy-proof mechanism is Li (2017)'s ascending clock auction. A bidder watches a price rise and decides only whether to stay in or drop out. She never reasons about other bidders, and she never asks what a different bid today would imply for tomorrow. The mechanism we study is the reallocation counterpart. Indivisible goods, called objects, lie on a line. An agent moves step by step along that line and, at each step, decides only whether to stop where she is or continue. Indeed, the Crawler of Bade (2019) admits an obviously strategy-proof implementation in which an agent observes only the object she currently holds and the neighboring available object in her direction of movement, and is never told any other agent's choices (Proposition 4 in Appendix D).[1] This theoretical result sharpens our behavioral hypothesis. If a mechanism can be presented as a sequence of two-option decisions that requires no reasoning about anyone else, agents may be able to navigate it even when they cannot navigate the same allocation rule in direct form.

We study an object-reallocation problem with five agents and single-peaked preferences. Each agent initially owns one object. Objects are ordered on a line, and preferences are single-peaked with respect to that order. Ordered objects with single-peaked preferences describe a range of settings in which reallocation is common and usually decentralized.[2]

[1]The parallel with the clock auction is not exact in one respect. Li's clock is common to all bidders and rises exogenously. Here each agent advances along the line at her own pace, chooses her direction at the first stage, and advances only when the holder of the neighboring object agrees to trade.

[2]Examples include offices or dormitory rooms ordered by size, parking spaces ordered by distance

We compare TTC (Shapley and Scarf, 1974) and the Crawler (Bade, 2019) under static and gradual implementations.[3] Both rules are efficient, individually rational, and strategy-proof on this domain. The Crawler admits an obviously strategy-proof implementation; TTC does not, with five agents.[4]

Our experiment uses a 2×2 between-subjects design: Static TTC, Static Crawler, Gradual TTC, and Gradual Crawler. In the static treatments, subjects submit a complete (single-peaked) ranking and the rule is applied. In the gradual treatments, subjects choose an action stage by stage as the mechanism unfolds. Only the Gradual Crawler is obviously strategy-proof.

A key feature of the design is that the treatments differ in how the mechanisms are implemented, not in the allocation they produce under truthful reporting. We use the same 20 preference–endowment environments in all treatments, constructed so that all four implementations select the same allocation when subjects report truthfully. Differences in realized welfare therefore cannot be attributed to differences in the mechanisms' truthful outcomes; they arise from behavior.

The Gradual Crawler attains the highest efficiency of the four treatments and the highest rate of truthful first choices. Two comparisons help identify the source. Holding the rule fixed and varying the implementation, the Gradual Crawler outperforms the Static Crawler, so the Crawler rule alone cannot explain the result. The corresponding comparison for TTC points in the opposite direction. The Gradual TTC performs worse than the Static TTC, so gradual implementation alone cannot explain it either. A third potential channel is constrained by the design. Bó and Hakimov (2020) attribute the gains from iterative deferred acceptance largely to feedback within a round: a student learns that her own application was rejected and adjusts. In our gradual mechanisms the analogous own-outcome signal, learning that one's chosen object has been assigned to someone else, is present in both implementations in equal measure, yet they are the best and the worst performers, so this feedback cannot account for the difference between them. Beyond it, neither implementation conveys any information about any individual opponent's choice (Proposition 2). What distinguishes the Gradual Crawler is the structure of the decision problem: a sequence of stop-or-continue choices, with at most three options at the first stage and two thereafter.

The evidence is mixed, and we do not present it as stronger than it is. The Gradual

from an entrance, seats ordered by row, and perhaps the leading case, work shifts, clinic appointments, or class sections ordered by time of day, where each agent has a preferred hour and likes a slot less the farther it is from that hour.

[3] Throughout, *gradual* refers to our own implementations, in which the mechanism unfolds over stages and agents choose as it does. We reserve *sequential*, *dynamic*, and *iterative* for the corresponding mechanisms of other authors, which we cite under the names they use, and for the sequential questioning of agents one at a time, as in the original implementation of Bade (2019).

[4] On the single-peaked domain, TTC is OSP-implementable if and only if there are at most three agents (Bade, 2019); on the unrestricted domain, no obviously strategy-proof implementation of TTC exists with three or more agents (Li, 2017).

Crawler produces the highest rate of truthful first choices but not the highest rate of fully truthful behavior over the entire decision problem. Violations of individual rationality are rare in all treatments and do not differ significantly across them. The effect of obvious strategy-proofness in our data is concentrated on early, welfare-relevant decisions rather than being spread evenly across all decision points. In this respect, our findings are consistent with an interpretation of obvious strategy-proofness in Brown et al. (2026): the behavioral performance of an obviously strategy-proof mechanism depends on the decision problem created by its implementation, not only on its formal incentive properties.

The paper makes three contributions. First, a design result: the Crawler admits an obviously strategy-proof implementation in which each agent's information consists of the set of available objects together with her own current holding and her own past choices (Proposition 2), and a still less informative implementation in which she is shown only her current holding and the nearest available object in her direction (Proposition 4). Bade (2019)'s original construction questions agents one at a time, so being asked to choose is itself informative about specific other agents. Simultaneous choices within a stage remove this. Second, experimental evidence that the implementation format affects realized welfare even when the allocation under truthful reporting is held fixed, with the 2×2 design separating the rule from the implementation format. Third, evidence on the source of this effect. Feedback cannot account for it. The own-outcome feedback that Bó and Hakimov (2020) identify is common to both gradual implementations, which are the best and worst performers, and no information about any individual opponent is ever conveyed (Proposition 2). Gradual unfolding by itself cannot account for it, because the Gradual TTC performs worse than its static counterpart. What remains consistent with our data is the sequence of safe stop-or-continue decisions that obvious strategy-proofness formalizes, although the experiment does not isolate this structure as the only possible channel.

1.1 Related Literature

A recurring finding is that strategy-proofness is valuable but not behaviorally sufficient. In an early experiment on on-campus housing, Chen and Sönmez (2002) compare random serial dictatorship with squatting rights and TTC. TTC is theoretically superior and achieves higher efficiency in the laboratory, yet truthful reporting is imperfect under both. In school choice, Chen and Sönmez (2006) compare the Boston, Gale–Shapley, and TTC mechanisms.[5] They document extensive manipulation under Boston, but also find that Gale–Shapley outperforms TTC in their environment, contrary to the theoretical efficiency ranking. Subsequent work shows that information, beliefs, and experience

[5]The Boston mechanism is also known as immediate acceptance, and Gale–Shapley as deferred acceptance.

matter: Pais and Pintér (2008) and Pais et al. (2011) vary the information available to subjects; Zhu (2015) studies the transmission of experience; Chen and He (2021) study information acquisition and provision. Field evidence points the same way. Hassidim et al. (2016), Rees-Jones (2018), and Shorrer and Sóvágó (2018) document deviations from truthful or undominated reporting in high-stakes centralized markets, and Chen and Pereyra (2019) show that students may omit selective schools from their reports even under a strategy-proof mechanism.

A second line of research studies mechanisms that unfold over time. Echenique et al. (2016) study a dynamic implementation of deferred acceptance and find frequent skipping of potential partners, with only about half of outcomes stable. Klijn et al. (2019) compare static and dynamic deferred acceptance in school choice. Gong and Liang (2025) study a dynamic mechanism used in Chinese college admissions and show that it can be more robust to complexity than deferred acceptance and Boston.

Closest to us is Bó and Hakimov (2020), who compare standard deferred acceptance with two iterative versions in which students apply one step at a time. In one, a student learns only whether her own application was rejected; in the other, universities' cutoffs are announced after each step. Both produce more stable outcomes and more straightforward behavior than the standard mechanism, and the authors attribute the larger part of the gain, which comes from making the mechanism iterative rather than from the cutoffs, to the feedback within the round that lets a student discover that a deviation has failed. They set obvious strategy-proofness aside as an explanation, since straightforward play in their iterative mechanisms is not even a dominant strategy.

We share the broad idea that unfolding a mechanism over time can improve behavior, but differ in three respects. We study object reallocation rather than school choice. We vary the allocation rule as well as the implementation format, whereas their design holds the rule fixed and varies the format and the information released. And because our two rules do not generally agree, we construct the 20 environments so that all four treatments select the same allocation under truthful reporting, which gives a single welfare benchmark for comparisons across rules as well as across formats. Our gradual mechanisms also occupy a different informational position. No agent is ever told any individual opponent's choice (Proposition 2), as in their no-cutoff version. The set of still-available objects that we display is an anonymous summary of what others have done, playing a role loosely analogous to their cutoffs. Proposition 4 shows that the Crawler can be run without even that. What matters for the comparison is that the own-outcome feedback their channel relies on is present in both of our gradual implementations, which are the best and the worst performers, so it cannot explain the difference between them.

We are also close to Bó and Hakimov (2024), who introduce pick-an-object mechanisms, in which agents choose objects sequentially from menus, may be rejected, and then choose again from smaller menus. Their experiment compares direct, pick-an-object, and obviously strategy-proof implementations of TTC and serial dictatorship, finding that the latter two often induce behavior closer to theoretical predictions. Their paper develops a broad class of mechanisms implementing familiar rules. We instead fix two rules in a single-peaked reallocation environment and vary only the implementation, holding the truthful outcome fixed. This allows us to show that gradual implementation by itself is not enough.

Li (2017) introduces obvious strategy-proofness and provides experimental evidence that subjects follow dominant strategies more often under obviously strategy-proof mechanisms than under strategically equivalent alternatives.[6] Brown et al. (2026) study a rationing problem and compare a direct mechanism, an obviously strategy-proof mechanism, a sequential mechanism, and one with nonbinding real-time feedback. The obviously strategy-proof mechanism performs similarly to its direct counterpart, while the feedback mechanism performs better. Our findings are consistent with this view: obvious strategy-proofness does not improve every behavioral outcome in our data, but the obviously strategy-proof implementation improves the outcomes that matter most for welfare. Relatedly, Katuščák and Kittsteiner (2025) show that making contingent reasoning easier raises truthful reporting, and Nagel and Saitto (2026) propose a measure of strategic complexity for strategy-proof mechanisms. Because all four of our implementations are strategy-proof while only one is obviously strategy-proof, this measure lets us compare all four. We compute it for our four mechanisms in Online Appendix A, where we also discuss how the resulting ranking relates to our data.

The object-reallocation problem itself is introduced by Shapley and Scarf (1974). On the single-peaked domain, Bade (2019) introduces the Crawler and shows that it is efficient, individually rational, and obviously strategy-proof; see also Schummer and Serizawa (2019). Her implementation has perfect information and questions agents one at a time. Our Propositions 2–4 show that the logic of the Crawler is compatible with far less information than questioning agents one at a time conveys.

The rest of the paper proceeds as follows. Section 2 introduces the model, the two rules, the four implementations, and our theoretical results. Section 3 describes the experimental design and hypotheses. Section 4 presents the results. Section 5 asks whether feedback or strategic complexity can account for them. Section 6 concludes. Appendices A–H contain formal definitions, the extensive-form framework, the proofs omitted from Section 2, the analysis of the information conveyed by each implementation,

[6]Obvious strategy-proofness is one of several formalizations of simplicity, which differ in what they ask of agents. Börgers and Li (2019) call a mechanism strategically simple if optimal choices can be based on first-order beliefs about the others' preferences alone, so that higher-order beliefs are irrelevant. Every dominant-strategy mechanism is strategically simple, so all four of our implementations are, and that notion does not separate them. Pycia and Troyan (2023) instead vary how far ahead an agent plans and obtain a family of simplicity standards, with obvious dominance one member and the more demanding strong obvious dominance another. The measure of Nagel and Saitto (2026) that we use below is closer in spirit to the latter approach, in that it ranks rather than distinguishes.

and supporting tables. The Online Appendix contains the analysis of strategic complexity, the preference profiles used in the experiment, and the experimental instructions.

2 Theoretical Framework

2.1 The Model

There are five agents, $N = \{1, \ldots, 5\}$, and five objects, $O = \{o_1, \ldots, o_5\}$. Objects are arranged on a line in the order $o_1, \ldots, o_5$ (increasing index), and agent i is endowed with object o_i. Each agent has strict preferences over objects that are *single-peaked* with respect to this line: each agent has a most-preferred object, her *peak*, and on each side of the peak objects become less attractive as they lie farther from it. How an agent ranks two objects on opposite sides of her peak is unrestricted. Let $\mathcal{P}$ denote the set of strict single-peaked preferences and $P = (P_1, \ldots, P_5) \in \mathcal{P}^5$ a preference profile. An *allocation* assigns each agent one object and each object to one agent. We denote by $\mathcal{X}$ the set of allocations, and a *(static) mechanism* is a mapping $\varphi : \mathcal{P}^5 \to \mathcal{X}$.

A mechanism is *efficient* if it never selects a Pareto-dominated allocation, *individually rational* if each agent weakly prefers her assignment to her endowment, and *strategy-proof* if no agent can obtain a better assignment by misreporting her preferences. Formal definitions are in Appendix A.

2.2 Obvious Strategy-Proofness

Strategy-proofness guarantees that truthful reporting is optimal whatever others do. Yet laboratory evidence often documents deviations even when truthful reporting is a weakly dominant strategy, suggesting that such incentive guarantees may be difficult to recognize and apply in practice. Obvious strategy-proofness (Li, 2017) strengthens strategy-proofness by requiring the optimality of truthful behavior to be transparent.

The concept applies to mechanisms viewed as extensive game forms. Consider any point at which an agent chooses an action and any deviation from truthful behavior that begins at that point. The agent compares the worst possible outcome from taking the truthful action with the best possible outcome from deviating. Obvious strategy-proofness requires that the former be at least as good as the latter. Equivalently, truthful behavior is safe without requiring any reasoning about the unobserved choices of others or about future play.

The formal definition is given in Appendix B. It allows for imperfect information, as required by the simultaneous choices within each stage of our gradual mechanisms, and follows Li (2017).[7] Strictly speaking, obvious strategy-proofness is a property of an

[7]The perfect-information formulation of Arribillaga et al. (2020) and Arribillaga et al. (2023) is a

implementation rather than of a rule. Following standard usage, we say that a rule is *obviously strategy-proof*, or *OSP-implementable*, if it admits an obviously strategy-proof implementation. Likewise, we say that a particular implementation is obviously strategy-proof if truthful behavior is obviously dominant in that implementation.

2.3 Two Rules: Top Trading Cycles and the Crawler

The first rule we consider is TTC (Shapley and Scarf, 1974). Fix a preference profile. TTC proceeds iteratively:

1. Each remaining agent points to the owner of the object she most prefers among the remaining objects.
2. At least one cycle forms, possibly several. In each cycle, each agent receives the object owned by the agent she points to.
3. Agents in cycles leave with their assigned objects.

Repeat with the remaining agents and objects until all agents are assigned.

The second rule is the Crawler (Bade, 2019, Schummer and Serizawa, 2019). Fix a preference profile. The Crawler proceeds iteratively:

1. Each remaining agent who most prefers her current endowment among the remaining objects keeps it and leaves. Repeat until no such agent remains.
2. Among the remaining agents, identify pairs whose current endowments are adjacent and who each prefer the other's current endowment to her own. At least one such pair exists, possibly several. In each such pair the two agents exchange their current endowments.
3. The remaining agents continue with their possibly updated endowments.

Repeat until all agents are assigned.

We summarize the known properties of the two rules. Some are specific to the single-peaked domain, while others hold on general domains.

Theorem (Shapley and Scarf, 1974; Roth and Postlewaite, 1977; Roth, 1982; Bade, 2019)**.** TTC is efficient, individually rational, and strategy-proof. Moreover, on the single-peaked domain, TTC is OSP-implementable if and only if there are at most three agents.

Theorem (Bade, 2019)**.** On the single-peaked domain, the Crawler is efficient, individually rational, and obviously strategy-proof.

special case. For the question of whether a rule admits an obviously strategy-proof implementation, restricting attention to perfect information is without loss of generality (Ashlagi and Gonczarowski, 2018, Mackenzie, 2020). We use the more general definition because we verify obvious dominance for particular implementations in which agents choose simultaneously.

2.4 Four Implementations

We study four implementations: the static versions of TTC and the Crawler, and their gradual counterparts.

In the static implementations, each agent submits a ranking of the objects, restricted to be single-peaked, to a central authority, which then applies either TTC or the Crawler to the submitted profile. We refer to the resulting mechanisms as the Static TTC and the Static Crawler.

The Gradual TTC unfolds over stages. In the first stage, all agents choose one object.[8] As in TTC, at least one cycle forms. Each agent in a cycle is assigned the object she chose and leaves. In each subsequent stage, consider the agents who remain. If the object an agent chose in the previous stage is still available, she takes no action and keeps that choice. Otherwise she chooses one of the adjacent available objects, that is, the available objects immediately before or after her previous choice in index order. As in TTC, at least one cycle forms, and each agent in a cycle is assigned the object she chose and leaves. The procedure continues until all agents are assigned.[9]

The Gradual Crawler unfolds over stages in the same way. In the first stage, all agents choose either their current endowment or one of its adjacent objects. If an agent chooses her current endowment, she keeps it and leaves. If two agents choose each other's objects, they exchange. At the end of the stage, the assignment of each agent still in the procedure becomes *final* if no further move in her direction is possible. An agent moving toward larger-indexed objects receives a final assignment if and only if the object she holds has the largest index among the objects still available. Similarly, an agent moving toward smaller-indexed objects receives a final assignment if and only if the object she holds has the smallest such index.[10] Otherwise the assignment is provisional and the agent remains in the procedure holding the new object. In each subsequent stage, consider the agents who remain. If an agent did not exchange in the previous stage and the object she chose is still available, she takes no action and keeps that choice. Otherwise she chooses again, continuing in the same direction as before. If her previous choice was toward a

[8] Choosing an object can be read as pointing to the owner of the most preferred object.

[9] Our Gradual TTC is strategy-proof. Fix an agent and the strategies of all others. Because agents do not observe one another's choices, the others' strategies cannot condition on this agent's choices directly, only on the evolution of the set of available objects. While the agent remains in the procedure, her own choices do not affect that set: objects are removed only when assigned through cycles, and any cycle affected by her choice must include her, in which case she is assigned and leaves. Hence, before she is assigned, the sequence of available-object sets, and therefore the behavior of the others, is independent of her choices. Conditional on each available-object set her problem is TTC on the corresponding subproblem, so choosing the most preferred available object whenever a new choice is required is weakly optimal.

[10] Whether an assignment is final is checked at the end of the stage. If making one agent's assignment final leaves another remaining agent's object extreme in that agent's direction of movement, the latter assignment becomes final as well, and the check is repeated until no further assignment becomes final. Lemma 1 shows that the resulting assignments do not depend on the order in which these checks are carried out.

smaller-indexed object she may choose either her current object or the nearest available object with a smaller index, and symmetrically in the other direction. If she chooses her current object, she keeps it and leaves. If two agents choose each other's objects, they exchange, and whether an assignment is final is determined exactly as in the first stage. The procedure continues until all agents are assigned.

In both gradual mechanisms, all agents called upon within a stage choose simultaneously. At the beginning of each stage an agent observes the set of objects still available and the object she currently holds. Agents do not observe the individual choices of other agents, either during or after a stage. Others' choices are conveyed only indirectly, through the evolution of the set of available objects. This structure is reflected in the information sets of the extensive game forms in Appendix B.

We now fix notation and state a fact about the Gradual Crawler that will be used throughout the rest of the section. Let G denote the extensive game form associated with the Gradual Crawler. For a history h at the beginning of a stage, let $O^h \subseteq O$ denote the set of objects still available, $N^h \subseteq N$ the set of agents still in the procedure, and $e^h : N^h \to O^h$ the one-to-one mapping assigning each agent her *current endowment*, where $e^h(i)$ is the object held by agent i at h. Let $\sigma_i(h)$ denote the sequence of agent i's past choices. This sequence is empty at the initial history.

An agent who does not choose her endowment at stage 1 moves toward either a larger- or a smaller-indexed object, and her direction of movement is fixed by that choice; we write $d_i \in \{+, -\}$ for it. An agent who chooses her endowment at stage 1 keeps it and leaves the procedure, so at any history h with stage 2 or onward every agent in N^h has a well-defined direction.

For $O' \subseteq O$, $o \in O'$, and $d \in \{+, -\}$, write

$$\text{adj}(o, d, O') = \begin{cases} \min\{o' \in O' : o' > o\} & \text{if } d = +, \\ \max\{o' \in O' : o' < o\} & \text{if } d = -, \end{cases}$$

for the object adjacent to o in O' in direction d, with $\text{adj}(o, d, O') = \emptyset$ when no such object exists. The rule in footnote 10 can then be stated as follows. The assignment of an agent i who remains in the procedure becomes final when $\text{adj}(e(i), d_i, O') = \emptyset$, where O' is the set of objects still available.

Lemma 1. Let h be a history at the beginning of a stage, and suppose the choice of every agent in N^h is fixed. We say that $i, j \in N^h$ *exchange* if $e^h(i)$ and $e^h(j)$ are adjacent in O^h and each chooses the other's current endowment.[11] Let $\mu : N^h \to O^h$ be the assignments

[11]Since endowments are distinct and a choice selects a single object, each agent has at most one such partner.

that result from the exchanges of the stage,

$$\mu(i) = \begin{cases} e^h(j) & \text{if } i \text{ exchanges with } j, \\ e^h(i) & \text{otherwise,} \end{cases}$$

which is one-to-one.

Let h' denote the history at the beginning of the next stage. Then the set of agents assigned during the stage, together with the object assigned to each of them, is uniquely determined. In particular, it does not depend on the order in which the checks of footnote 10 are carried out. Hence $O^{h'}$ and $N^{h'}$ are well defined, and for each $i \in N^h$,

$$i \text{ is assigned during the stage} \iff \mu(i) \notin O^{h'}. \tag{1}$$

If i is assigned during the stage, then i receives $\mu(i)$; otherwise, $e^{h'}(i) = \mu(i)$.

The proof is in Appendix C.

Corollary 1. For each i and each history h at which i is called upon at a stage $k \geq 2$, the set of actions available to i is

$$A(h) = \{e^h(i),\ \mathrm{adj}(e^h(i), d_i, O^h)\},$$

which depends only on $e^h(i)$, d_i, and O^h. At stage 1 the action set depends only on i's endowment.

The rule that makes an assignment final is applied repeatedly within a stage, and one might worry that the outcome depends on the order in which agents are checked. It does not. The set of objects assigned during a stage is the same however the checks are ordered, so the set of objects still available at the start of the next stage is a well-defined description of where the procedure stands. Two consequences follow. The mechanism is a well-defined game form, and by Corollary 1, each agent's two options at any stage after the first are determined by just her own current holding, her direction, and the set of available objects.

2.5 The Gradual Crawler Is Obviously Strategy-Proof

We begin with a simple consequence of single-peakedness that we use twice: first to describe truthful behavior, and again in Appendix D to show that this description requires almost no information.

Lemma 2. Let P_i be single-peaked, let $O' \subseteq O$, let $o \in O'$, and let $d \in \{+, -\}$ with $o'' = \mathrm{adj}(o, d, O') \neq \emptyset$. Let $F = \{o\} \cup \{o' \in O' : o' \text{ lies strictly in direction } d \text{ from } o\}$. Then the most preferred object in F according to P_i differs from o if and only if $o'' \, P_i \, o$.

The proof is in Appendix C. An agent deciding whether to continue faces what looks like a global question: is the best object I can still reach ahead of me? Under single-peaked preferences, the answer coincides with that of a local question: do I prefer the neighboring available object to the one I am currently holding? The two questions cannot come apart. Thus an agent who compares only her current object with the neighboring available object behaves as if she had evaluated the whole set of objects she could still reach.

Proposition 1. Truthful behavior is obviously dominant in the Gradual Crawler. Hence the Gradual Crawler is an obviously strategy-proof implementation of the Crawler.

The proof is in Appendix C. The mechanism turns the allocation problem into a sequence of local decisions. At each stage, an agent decides whether to stop where she is or continue along the line. Under single-peaked preferences, this decision has a simple logic. As long as continuing brings her closer to the best object she can still reach, continuing is safe. She moves toward her peak whatever the others do, and stopping early would leave her short of objects closer to her peak. Once continuing would carry her past her peak, the comparison reverses, and stopping is safe because moving further leads only to less preferred objects. She never compares many rankings or traces the possible continuations of the mechanism, and by Lemma 2 the local comparison she makes gives the same answer as the global one.

Remark 1. Truthful behavior is not obviously dominant in the other three implementations. For the Gradual TTC, Example 1 below illustrates an earliest point of departure at which the best outcome following a deviation (o_2) is strictly preferred to the worst outcome following the truthful action (o_1). For the two static mechanisms the earliest point of departure of any two distinct reporting strategies is the single reporting stage, so it suffices to show one preference relation and one admissible misreport for which the comparison fails. Consider agent 1 with endowment o_1 and peak o_3, and the single-peaked misreport that ranks o_2 first and o_3 second. Under truthful reporting, if every other agent ranks her own endowment first, agent 1 receives o_1 under both Static TTC and Static Crawler. Under the misreport there are reports of the others for which she does strictly better. Under Static TTC, if agent 2 ranks o_2 first and agent 3 ranks o_1 first, agent 1 receives o_3. Under Static Crawler, if agent 2 ranks o_1 first, agents 1 and 2 exchange and agent 1 receives o_2.

It is worth seeing why obvious dominance fails in the Gradual TTC. TTC is strategy-proof, so truthful reporting is optimal. But a report affects which cycle forms, which agents leave, and which objects remain available later. Whether a deviation hurts can therefore depend on how the others point and on which cycles would form under alternative reports. To see that truthful reporting is optimal, an agent may have to compare several possible continuations.

Example 1. Suppose agent 1 most prefers o_3, then o_2, then o_1, so her truthful first choice is o_3. Suppose agent 2 chooses o_3 and agent 3 chooses o_2. Then agents 2 and 3 form a cycle and leave, and agent 1 receives o_1. So the truthful first choice can lead to o_1. Now consider another situation. Agent 2 again chooses o_3, but agent 3 chooses o_1. If agent 1 deviates and chooses o_2, the three agents form a cycle, agent 1 chooses agent 2's object, agent 2 chooses agent 3's, agent 3 chooses agent 1's. Thus agent 1 receives o_2. The best outcome following a deviation at stage 1 is therefore strictly better than the worst outcome following the truthful first choice, since agent 1 prefers o_2 to o_1.[12]

Two further sets of theoretical results support the interpretation of the experiment. We develop them in the appendices and use them in Section 5. Appendix D shows that neither gradual implementation ever tells an agent what any particular other agent chose, and that the Crawler can even be run as a two-option procedure in which an agent is shown only the object she holds and the nearest available object in her direction (Propositions 2–4). Online Appendix A computes the strategic complexity of all four implementations under the measure of Nagel and Saitto (2026) (Table 4), which, unlike obvious strategy-proofness, ranks all four.

3 Experimental Design

3.1 Treatments and Procedures

We study four treatments in a between-subjects design: Static TTC, Static Crawler, Gradual TTC, and Gradual Crawler (Table 1). For each treatment we ran two sessions with 25 subjects per session, giving 50 subjects per treatment and 200 in total.[13] Each problem had five agents and five objects, and subjects were randomly rematched into groups of five each round.

[12] The example shows only that our particular Gradual TTC implementation is not obviously strategy-proof. That TTC admits no obviously strategy-proof implementation with five agents on this domain is due to Bade (2019); see the theorem in Section 2.3.

[13] The sample size was set by power calculations based on a small pilot. The primary comparisons test whether the Gradual Crawler achieves higher efficiency than each of the other three treatments, using Mann–Whitney tests. The calculations treat outcomes across rounds as independent, which provides a lower bound on the required number of rounds, and use $\alpha = 0.05$ and power $1-\beta = 0.8$. The lower bound was 190 group-round observations; we rounded up to 200 and planned 20 rounds with 10 groups of five per treatment. We return to the implications of running only two sessions per treatment in footnote 16 in Section 4.1.

Table 1: 2×2 experimental design.

	Top Trading Cycles (TTC)	**Crawler**
Static implementation	Static TTC	Static Crawler
Gradual implementation	Gradual TTC	Gradual Crawler *(obviously strategy-proof)*

At the beginning of each session, subjects read the instructions and completed a quiz to confirm that they understood the procedure.[14] The instructions stated that subjects would be randomly rematched each round and that preferences and endowments would be assigned at random.

Each session lasted 20 rounds, each round a new problem. We constructed 20 preference–endowment environments and used the same 20 in all four treatments (Online Appendix B). In each environment, under truthful reporting TTC and the Crawler select the same allocation, and no subject is assigned her endowment. Preferences and endowments were assigned at random each round, and rounds were independent. Subjects observed only their own preferences and endowment. After each round they learned only the object assigned to them. At the end of the experiment one round was drawn at random for payment.

Payments depended on the object assigned: 2,250 yen for the most preferred object, 1,750 for the second, 1,250 for the third, 750 for the fourth, and 250 for the least preferred, plus a participation fee of 750 yen. Sessions lasted 40 minutes on average and average earnings were 2,500 yen, about 16 USD at the time.[15]

3.2 Outcome Measures

We evaluate outcomes using subjects' true preferences. In each round each subject's welfare from her assigned object is measured by a pre-specified rank-based score, and aggregate welfare is the sum of these scores within a group. Our main measure is the *Pareto efficiency ratio*,

$$\text{Pareto efficiency ratio} = \frac{\text{welfare achieved in the round}}{\text{welfare under truthful reporting in that round}}.$$

[14]See Online Appendix C for English translations of instructions, quiz, and decision screens.

[15]The experiment was programmed in oTree (Chen et al., 2016). Subjects were University of Osaka students registered in ISER's subject pool, managed with ORSEE (Greiner, 2015). Balance checks are in Appendix E: treatments are balanced by gender, and mean age is close to 23 in all treatments, with Static TTC subjects significantly younger by less than two years, probably due to the exam schedule differences during the period in which the experimental sessions were run.

Because the allocations selected under truthful reporting coincide across the four mechanisms by construction, a single benchmark applies to all treatments. That allocation, though Pareto efficient, need not maximize the aggregate rank-based score among all allocations, so the ratio can in principle exceed 100% in a given group-round.

We also report a *maximum Pareto efficiency* measure, which divides aggregate welfare by the highest aggregate welfare attainable across all individually rational allocations for the truthful reportings. This measure is bounded above by 100% for individually rational outcomes.

Two behavioral measures complete the set. *Truthful first choice* is an indicator equal to one if a subject's first active decision is consistent with truthful behavior: in the Gradual Crawler, moving in the correct direction, that is, toward the object adjacent to her endowment that is closer to her peak; in the Gradual TTC, choosing her peak; in the static mechanisms, ranking her peak first. *Fully truthful behavior* equals one if behavior is truthful at every decision point: in the gradual mechanisms, moving toward the most preferred object she can still reach at every stage; in the static mechanisms, submitting a ranking that coincides with her true preferences. Finally, a *violation of individual rationality* is an outcome in which a subject receives an object worse than her endowment according to her true preferences.

3.3 Hypotheses

Hypothesis 1. The Gradual Crawler yields higher welfare than each of the other three treatments.

Hypothesis 1 is our primary, pre-registered prediction, stated in terms of the efficiency measures of Section 3.2.

Hypothesis 2. Truthful behavior in the first choice is more frequent in the Gradual Crawler than in each of the other three treatments.

The first choice is a simple and comparable measure of whether subjects follow the basic incentive logic of each mechanism. In the gradual mechanisms it is the first active decision, taken before later choices, changing feasibility, or the actions of others can further shape the problem. The measure is less comparable across treatments than the efficiency ratios, because a truthful first choice requires different amounts of reasoning in different mechanisms; Hypotheses 2 and 3 are therefore secondary.

Hypothesis 3. Fully truthful behavior is more frequent in the Gradual Crawler than in each of the other three treatments.

Hypothesis 4. Subjects are less likely to receive an object worse than their endowment according to their true preferences in the Gradual Crawler than in each of the other three treatments.

4 Results

4.1 Efficiency

Figure 1 and Table 2 report efficiency across treatments. The Pareto efficiency ratio was the pre-registered measure; as a robustness check we also report the maximum Pareto efficiency measure. We compute averages in two ways: *overall* efficiency sums welfare across all groups and rounds in a treatment and divides by the benchmark, while *group-round* efficiency averages efficiency across group-round observations. The Gradual Crawler achieves the highest efficiency on every measure reported.

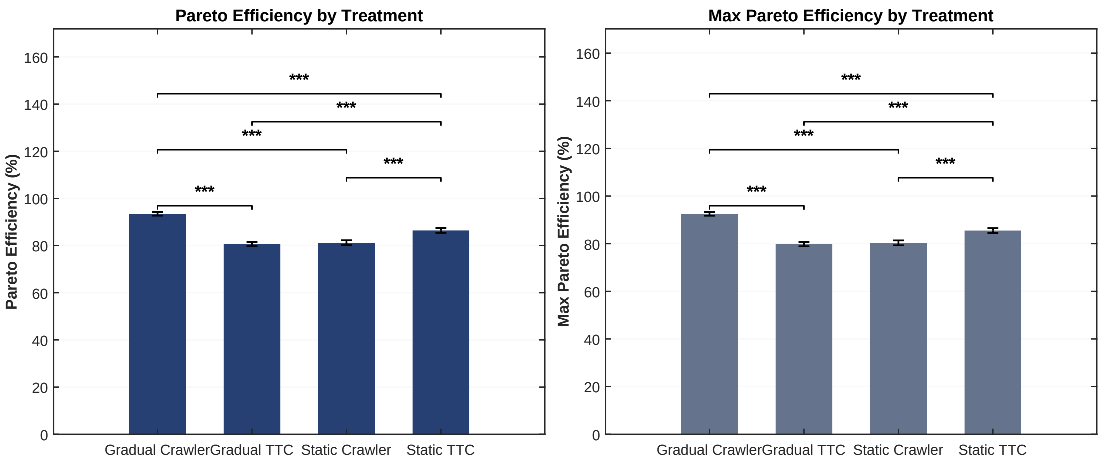

Figure 1: Efficiency across treatments.

Table 2: Efficiency across mechanisms.

	Pareto efficiency (%)		Max. Pareto efficiency (%)	
Treatment	Overall	Group-round avg.	Overall	Group-round avg.
Gradual Crawler	93.33	93.47	92.41	92.53
Gradual TTC	80.45	80.65	79.66	79.83
Static Crawler	81.02	81.19	80.22	80.33
Static TTC	86.27	86.41	85.42	85.51

We test for differences using Mann–Whitney tests with the Bonferroni correction for multiple comparisons.[16] Pairwise tests are in Appendix G, and Figure 1 illustrates the differences. Following Hypothesis 1 the comparisons of primary interest are between the Gradual Crawler and each of the other three mechanisms. The Gradual Crawler achieves significantly higher efficiency than each, at the 0.001 level or better, supporting Hypothesis 1.

[16] This test treats outcomes across rounds and groups as independent observations. We complement it with ordinary least squares regressions using standard errors clustered at the session level (Appendix H). The results are largely robust to the independence assumption and to the choice of test; the exception is the comparison between the Gradual Crawler and Static TTC, which loses statistical significance under session-level clustering.

Result 1. The Gradual Crawler yields the highest efficiency among the four mechanisms.

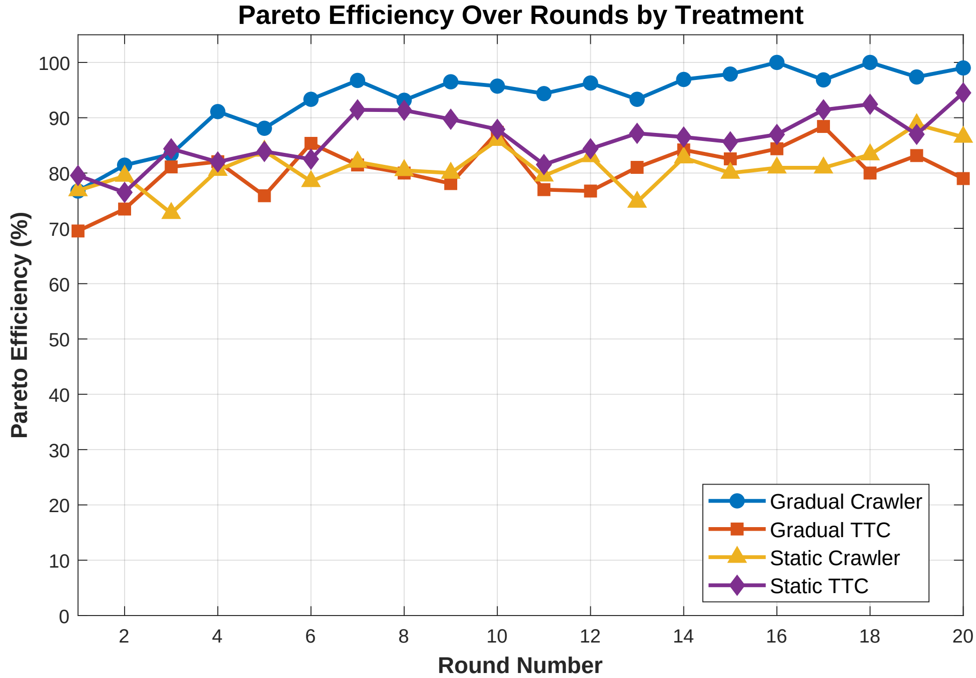

Figure 2: Pareto efficiency across rounds.

Figure 2 shows how efficiency evolves over the session. The four mechanisms perform similarly in the first three rounds with no clear ranking, which suggests initial learning. From round four onward the Gradual Crawler outperforms the others in every round and reaches 100% efficiency in several. No single preference profile stands out, so the relative performance of the mechanisms is not driven by a particular preference–endowment environment. Splitting the sample into rounds 1–10 and 11–20 (Appendix F), only the Gradual Crawler shows a significant increase.

4.2 Truthful Behavior

Table 3 reports truthful first choices, fully truthful behavior, and violations of individual rationality. The Gradual Crawler achieves the highest rate of truthful first choices. Mann–Whitney tests with the Bonferroni correction (Appendix G) show that it exceeds each other treatment at the 0.001 level or better, and regressions with standard errors clustered at the individual level (Appendix H) confirm this. Hypothesis 2 is supported.

Table 3: Truthful behavior and individual rationality.

Mechanism	Truthful first choice (%)	Fully truthful (%)	IR violations (%)
Gradual Crawler	94.20	62.60	0.30
Gradual TTC	52.30	52.30	0.10
Static Crawler	55.10	49.40	0.30
Static TTC	69.30	66.10	0.70

Result 2. The Gradual Crawler yields the highest frequency of truthful first choices among the four mechanisms.

The high rate of truthful first choices helps explain the efficiency result. Under the Gradual Crawler with single-peaked preferences, choosing the better of two neighboring objects moves a subject toward her most preferred object overall (Lemma 2), and the mechanism turns that task into a sequence of choices among at most three options.

On the stricter measure, Static TTC performs best. Appendix G shows that both Static TTC and Gradual Crawler significantly outperform the other two mechanisms, but the difference between them is not significant in the nonparametric tests, and the regression puts Static TTC ahead of the Gradual Crawler only at the 5% level. We therefore do not find strong support for Hypothesis 3.

Result 3. The Gradual Crawler does not yield the highest frequency of fully truthful behavior among the four mechanisms.

The gap between Results 2 and 3 matters for interpreting obvious strategy-proofness. The Gradual Crawler is the only obviously strategy-proof implementation here, and it performs best on welfare and on first choices, but not on truthfulness at every decision point. This is consistent with Brown et al. (2026), who find that the behavioral advantage of obvious strategy-proofness need not extend to every outcome measure. In our setting the Gradual Crawler improves the decisions most relevant for welfare rather than inducing truthful behavior throughout. One caveat about measurement applies. In the static treatments the submitted rankings must be single-peaked, so the set of possible untruthful reports is small. In the gradual treatments full truthfulness requires a truthful choice at every stage, giving more opportunities to be classified as untruthful. The measure is therefore not fully comparable across formats, which suggests caution in reading the strong performance of Static TTC on this dimension.

Turning to assignments below the endowment, Gradual TTC has the lowest violation rate and the Gradual Crawler the second lowest, but the pairwise tests in Appendix G show no significant differences after correction. Without correction the only significant difference is between the static and gradual versions of TTC at the 5% level. The regressions in Appendix H agree. Hypothesis 4 is not supported, plausibly because violations are rare everywhere, below 1% in all treatments.

Result 4. The Gradual Crawler does not significantly reduce violations of individual rationality relative to the other mechanisms.

5 Discussion

Experimental results presented in Section 4 support our primary hypothesis and show that the Gradual Crawler achieves the highest efficiency rates. According to the theoretical literature on obvious strategy-proofness, this superior performance is due to the fact that it is easy for participants to identify truthful reporting as an optimal course of action, since the Gradual Crawler is the only obviously strategy-proof mechanism among those tested. Could other explanations suggested in the literature be responsible for its superior empirical performance? In this section, we discuss two features proposed in the literature as contributing to the efficiency of mechanisms, feedback provision and strategic simplicity, and show that neither predicts the experimental performance of the mechanisms we study.

Feedback allows participants to learn new information and make inferences, which in turn may affect their behavior, particularly their truthful behavior. In fact, Bó and Hakimov (2020) attribute much of the gain from iterative deferred acceptance to feedback within a round. A student learns that her own application was rejected and abandons the failed deviation. Can this feedback channel explain our experimental results, with the Gradual Crawler outperforming the Gradual TTC? In Appendix D, we study the information transmitted by these gradual mechanisms. First, we show that the own-outcome feedback is present in both gradual implementations. In each mechanism, a subject learns when the object she chose has been assigned to someone else, yet the two sit at opposite ends of the efficiency ranking, and the one with a rejection-and-rechoose structure closest to theirs, the Gradual TTC, performs worst. What neither implementation conveys is any information about any individual opponent's choice (Propositions 2 and 3), and by Remark 3 the inference that does remain available is anonymous and present in both in equal measure, so richer social feedback is excluded as well. Second, we study the structure of the decision problem. The Gradual Crawler presents a sequence of stop-or-continue choices in which stopping is a natural default and, by Lemma 2, truthful behavior requires only a comparison between the object an agent holds and the neighboring available object. Proposition 4 shows that this is not merely an artifact of our interface: the mechanism can be implemented while revealing to an agent nothing beyond these two objects.

We study the strategic complexity of all four mechanisms in Online Appendix A, where we apply the measure of Nagel and Saitto (2026). This measure ranks strategy-proof mechanisms by how much contingent reasoning is required to recognize that truthful behavior is optimal. The resulting values cannot explain the experimental performance reported in Section 4. At $n = 5$ the measure assigns complexity 15 to each static mechanism and 5 to each gradual one (Table 4), so it separates the implementation formats but assigns the same complexity to the two gradual mechanisms, which sit at opposite ends of our efficiency ranking, and it ranks the Gradual TTC as simpler than both static mechanisms although it performs worst.

Table 4: Strategic complexity of the four mechanisms.

	Static TTC	Static Crawler	Gradual TTC	Gradual Crawler
General n	$2^{n-1}-1$	$2^{n-1}-1$	n	2 $(n=3)$; 3 $(n=4)$; 5 $(n=5)^{\dagger}$
$n=5$	15	15	5	5

†For $n \geq 6$ we establish the lower bound of 3 (Lemma OA.6 in Online Appendix A).

After the first stage the Gradual Crawler never displays more than two actions on any screen; the value of 5 arises at the level of continuation plans. A candidate for what separates the two gradual mechanisms is the property that obvious strategy-proofness formalizes. At every decision of the Gradual Crawler the truthful action is safe against everything that can still happen, whereas in the Gradual TTC even the first truthful choice is not (Example 1). Our evidence is consistent with this interpretation, though the experiment does not isolate the local choice structure as the only channel. Separating its components is a direction for further work.

6 Concluding Remarks

This paper provides experimental evidence on TTC and the Crawler in object-reallocation problems with single-peaked preferences, comparing static and gradual implementations of both. The Gradual Crawler, the only obviously strategy-proof implementation among the four, achieves significantly higher efficiency than the others and the highest rate of truthful first choices.

Our results suggest that a gradual implementation alone does not guarantee high efficiency. For TTC the gradual implementation performs worse than the static one. Nor does the Crawler rule alone account for the result, since the Gradual Crawler substantially outperforms the Static Crawler. The advantage appears to come from the combination of the rule with its gradual implementation, which produces a simple stop-or-continue procedure. And it cannot be attributed to feedback: information about other agents is never provided (Proposition 2 in Appendix D), and the own-outcome feedback emphasized by Bó and Hakimov (2020) is common to both gradual implementations, which are the best and the worst performers.

The theoretical part of the paper formalizes this idea. The Crawler admits an obviously strategy-proof implementation in which each agent is shown only the object she currently holds and the nearest available object in her direction of movement, and is never told any other agent's choice (Proposition 4 in Appendix D). In this sense the mechanism is the reallocation counterpart of an ascending clock auction: each agent advances along the line of objects and makes a single stop-or-continue decision at each step. That the whole task can be presented this way, and not merely that the mechanism happens to be obviously strategy-proof, may be part of what underlies its behavioral performance.

At the same time, our results suggest that obvious strategy-proofness is not a guarantee of truthful behavior at every decision point. The Gradual Crawler does not produce the highest rate of truthful behavior at every decision point, and violations of individual rationality do not differ significantly across treatments. This is in line with Brown et al. (2026), who find that an obviously strategy-proof mechanism does not outperform its direct counterpart in a rationing problem and that other forms of feedback can do better. Together these findings suggest that the behavioral performance of obvious strategy-proofness may depend on how the implementation presents decisions and on how mistakes at intermediate points affect final outcomes.

For practice, our findings suggest that a gradual implementation can raise welfare by creating a decision problem agents find easier to navigate, but that unfolding the mechanism over time is not by itself sufficient, and such implementations may take more steps to reach a final allocation. Designers may therefore wish to weigh the behavioral benefits of simplifying decisions against the cost of a longer procedure. Further theoretical and experimental work could study how different obviously strategy-proof implementations of the same rule differ in complexity, feedback, default options, and behavioral performance, and, in light of Proposition 4, how much information about the state of the procedure it is useful to display.

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Appendices

Appendix A collects formal definitions. Appendix B presents the extensive-form framework and the definition of obvious strategy-proofness used in the main text. Appendix C contains the proofs omitted from Section 2. Appendix D shows how much information each implementation conveys. Appendices E–H contain balance checks, tests for learning, pairwise tests, and regressions. Online Appendix A states the measure of strategic complexity of Nagel and Saitto (2026) formally and computes it for all four mechanisms. Online Appendix B contains the preference profiles used in the experiment, and Online Appendix C contains the English translations of the instructions, quizzes, and decision screens.

A Model and Definitions

There are five agents $N = \{1, \ldots, 5\}$ and five indivisible goods, called objects, $O = \{o_1, \ldots, o_5\}$. The objects are ordered by an exogenous linear order, and we index them so that this order coincides with their indices. Each agent i is endowed with o_i.

For each nonempty $O' \subseteq O$ and each pair $o_j, o_k \in O'$ with $j < k$, we say o_j and o_k are *adjacent in* O' if there is no ℓ with $j < \ell < k$ such that $o_\ell \in O'$.

Each agent i has strict preferences P_i over O that are *single-peaked* with respect to this order: there is a peak index $p_i \in \{1, \ldots, 5\}$ such that for each pair $j, k \in \{1, \ldots, 5\}$, if $k < j \leq p_i$ or $p_i \leq j < k$, then $o_j \, P_i \, o_k$. Thus o_{p_i} is agent i's most preferred object, and preferences fall strictly as one moves away from the peak on either side. Comparisons between objects on opposite sides of the peak are unrestricted. Let $\mathcal{P}$ denote the set of strict single-peaked preferences and $P = (P_1, \ldots, P_5) \in \mathcal{P}^5$ a profile. Let R_i denote the weak preference associated with P_i, so $o \, R_i \, o'$ if and only if $o \, P_i \, o'$ or $o = o'$.

An *allocation* is a one-to-one mapping $x : N \to O$. We write $x_i \equiv x(i)$, and let $\mathcal{X}$ denote the set of allocations. A (static) *mechanism* is a function $\varphi : \mathcal{P}^5 \to \mathcal{X}$. A mechanism φ is *efficient* if there is no $P \in \mathcal{P}^5$ and $x \in \mathcal{X}$ such that $x_i \, R_i \, \varphi_i(P)$ for each $i \in N$ and $x_i \, P_i \, \varphi_i(P)$ for some $i \in N$. It is *individually rational* if $\varphi_i(P) \, R_i \, o_i$ for each $P \in \mathcal{P}^5$ and each $i \in N$. Finally it is *strategy-proof* if $\varphi_i(P) \, R_i \, \varphi_i(P'_i, P_{-i})$ for each $P \in \mathcal{P}^5$, each $i \in N$, and each $P'_i \in \mathcal{P}$.

B Extensive Game Forms and Obvious Strategy-Proofness

This appendix states the definition of obvious strategy-proofness used in the paper. Because agents in our gradual mechanisms choose objects simultaneously within each stage, we cannot restrict attention to game forms with perfect information. Therefore we follow the general formulation of Li (2017), which permits imperfect information. Since all mechanisms here are deterministic we omit moves by nature, which simplifies notation without affecting any result.

B.1 Extensive Game Forms

An extensive game form with outcomes in $\mathcal{X}$ is denoted G. Let H be the set of histories, partially ordered by the precedence relation $\prec$, with initial history h_0 and terminal histories $Z \subseteq H$. At each non-terminal history h one agent $\tau(h) \in N$ moves and chooses from a nonempty set $A(h)$. Each terminal history $z \in Z$ yields an allocation $g(z) \in \mathcal{X}$.

To capture imperfect information, the non-terminal histories at which agent i moves are partitioned into *information sets*. Let $\mathcal{I}_i$ denote the collection of information sets of agent i, with typical element I_i. Agent i cannot distinguish histories in the same information set, so $A(h) = A(h')$ whenever $h, h' \in I_i$, and we write $A(I_i)$ for this common action set. Simultaneous choices within a stage are modeled in the usual way. Agents move one after another in an arbitrary order, and each agent's information set contains all histories that differ only in the choices made by the others in the same stage. We restrict attention to game forms with perfect recall and finite length. For a history h and an information set I_i, write $I_i \prec h$ if $h' \prec h$ for some $h' \in I_i$. Table B.1 collects the notation.

Table B.1: Notation for a generic extensive game form.

Object	Notation
Histories	H
Initial history	h_0
Terminal histories	Z
Agent who moves at h	$\tau(h)$
Information sets of agent i	$\mathcal{I}_i$
Actions available at I_i	$A(I_i)$
Allocation resulting from z	$g(z)$
Set of allocations	$\mathcal{X}$

A (pure) *strategy* for agent i assigns an action $s_i(I_i) \in A(I_i)$ to each $I_i \in \mathcal{I}_i$. Let S_i denote i's strategy set and $S = \prod_{i \in N} S_i$. For each $h \in H$ and each $s \in S$, let $z[h](s)$ denote the terminal history reached from h when s is followed thereafter. We write $z(s)$ for $z[h_0](s)$.

B.2 Obvious Dominance and OSP-Implementability

Fix $i \in N$ and $P_i \in \mathcal{P}$. A strategy $s_i \in S_i$ is *weakly dominant at* P_i if, for each $s'_i \in S_i$ and each $s_{-i} \in S_{-i}$, $g\big(z(s_i, s_{-i})\big)\ R_i\ g\big(z(s'_i, s_{-i})\big)$.

Obvious dominance strengthens this requirement at the information sets at which two strategies first differ. Given $s_i, s_i' \in S_i$, an information set $I_i \in \mathcal{I}_i$ is an *earliest point of departure* of s_i and s_i' if (i) $s_i(I_i) \neq s_i'(I_i)$; (ii) there is $s_{-i} \in S_{-i}$ with $I_i \prec z(s_i, s_{-i})$; and (iii) there is $s_{-i} \in S_{-i}$ with $I_i \prec z(s_i', s_{-i})$. Let $\alpha(s_i, s_i')$ denote the set of earliest points of departure.

A strategy $s_i \in S_i$ is *obviously dominant at* P_i if, for each $s_i' \in S_i$, each $I_i \in \alpha(s_i, s_i')$, each $h, h' \in I_i$, and each $s_{-i}, s_{-i}' \in S_{-i}$, $g\big(z[h](s_i, s_{-i})\big)\ R_i\ g\big(z[h'](s_i', s_{-i}')\big)$. In words, at any point where a deviation first takes effect, the worst allocation i can obtain by following s_i is at least as good as the best allocation she can obtain by deviating to s_i'. Because i may be unable to distinguish the histories in I_i, the comparison ranges over all histories in the information set as well as over all strategies of the others. An obviously dominant strategy is weakly dominant, so every obviously strategy-proof mechanism is strategy-proof (Li, 2017).

A mechanism $\varphi : \mathcal{P}^5 \to \mathcal{X}$ is *OSP-implementable* if there are an extensive game form G and a strategy profile $s(\cdot)$ such that for each $P \in \mathcal{P}^5$: (i) $s_i(P_i)$ is obviously dominant at P_i for each $i \in N$; and (ii) $g\big(z(s(P))\big) = \varphi(P)$. In that case we say φ is *obviously strategy-proof* and call G together with $s(\cdot)$ an obviously strategy-proof implementation of φ.[17]

C Proofs Omitted from Section 2

Proof of Lemma 1. Let $K \subseteq N^h$ be the set of agents who choose their own current endowment. Each is assigned that object and leaves, so the objects in $R_0 := \{\mu(i) : i \in K\}$ are removed. The agents in $N^h \setminus K$ remain in the procedure, agent i holding $\mu(i)$. Whether two agents exchange depends only on the choices made at this stage. Thus μ and K are fixed before any check is carried out. Subsequent checks can only remove additional objects.

The checks can therefore be represented by a mapping on sets of removed objects. For $R \subseteq O^h$ let

$$\Phi(R) = R \cup \big\{\mu(i) : i \in N^h \setminus K,\ \operatorname{adj}(\mu(i), d_i, O^h \setminus R) = \emptyset\big\},$$

so that $\Phi(R)$ adds to R the objects held by those agents whose provisional assignment the

[17]Our definition is consistent with the perfect-information formulation used elsewhere. For the question of whether a rule admits some obviously strategy-proof implementation, restricting attention to game forms with perfect information is without loss of generality, by the revelation-principle results of Ashlagi and Gonczarowski (2018) and Mackenzie (2020). The formulation of Arribillaga et al. (2020) and Arribillaga et al. (2023) is the corresponding special case. We require the general definition because we verify obvious dominance for particular implementations, the Gradual Crawler and the less informative form of Proposition 4, in which agents choose simultaneously within each stage, so the associated game forms have information sets containing more than one history.

check identifies as final once the objects in R have been removed. The mapping Φ has two properties. First, $R \subseteq \Phi(R)$. Second, Φ is monotone: if $R \subseteq R'$ then $O^h \setminus R' \subseteq O^h \setminus R$, and an object with no available object beyond it in $O^h \setminus R$ has none in $O^h \setminus R'$ either, so every element added at R is also added at R'.

Because $R \subseteq \Phi(R)$ for every R and O^h is finite, the chain $R_0 \subseteq \Phi(R_0) \subseteq \Phi^2(R_0) \subseteq \cdots$ is eventually constant. Let R^* denote its resulting set. Then $\Phi(R^*) = R^*$ and $R_0 \subseteq R^*$. Moreover R^* is the smallest set that contains R_0 and is a fixed point of Φ.[18]

We claim that every order of checking produces R^*. Carrying out the checks one at a time creates a chain $R_0 \subseteq R_1 \subseteq \cdots \subseteq R_q$, where each R_ℓ is obtained from $R_{\ell-1}$ by adding a single element of $\Phi(R_{\ell-1})$, and the process stops when $\Phi(R_q) = R_q$. Write $R := R_q$. On the one hand, R is a fixed point of Φ containing R_0, so $R^* \subseteq R$. On the other hand, $R \subseteq R^*$ by induction on ℓ. We have $R_0 \subseteq R^*$, and if $R_{\ell-1} \subseteq R^*$ then the element added at step ℓ lies in $\Phi(R_{\ell-1}) \subseteq \Phi(R^*) = R^*$. Hence $R = R^*$ regardless of the order of checking, and $O^{h'} = O^h \setminus R^*$ is well defined.

It remains to identify which agents leave the procedure and which objects they receive. By construction, every object that Φ adds is $\mu(i)$ for some $i \in N^h \setminus K$, so $R^* = \mu(K \cup F)$, where $F \subseteq N^h \setminus K$ is the set of agents whose assignment becomes final. Since μ is one-to-one, $\mu(i) \in R^*$ if and only if $i \in K \cup F$. An agent leaves the stage only by keeping her current endowment ($i \in K$) or by her assignment becoming final ($i \in F$). In either case she receives $\mu(i)$, which lies in R^* and hence not in $O^{h'}$. Every other agent remains, holding $\mu(i) \in O^{h'}$, so that $e^{h'}(i) = \mu(i)$. This is (1), and $N^{h'}$ is the set of agents who remain. □

Proof of Lemma 2. The restriction of a single-peaked preference relation to a subset of the line is again single-peaked. Every element of F other than o lies strictly in direction d from o. Thus o is an endpoint of F, and the restriction of P_i to F increases toward its peak and decreases thereafter. If the peak is o, then P_i is strictly decreasing along F away from o, so $o \, P_i \, o''$. Otherwise the peak is some $o^* \neq o$. Since o'' lies weakly between o and o^* in F, strict monotonicity on that side gives $o'' \, P_i \, o$. □

The proof of Proposition 1 uses the framework of Appendix B: the extensive game form associated with the Gradual Crawler, information sets I_i of agent i, earliest points of departure between two strategies, and the obvious-dominance requirement.

Proof of Proposition 1. Fix agent i with single-peaked preferences P_i. Whenever i is called upon, the available actions are as follows. At stage 1 she chooses her endowment or one of its adjacent objects, so there are three actions, or two if her endowment is o_1 or o_5. At every later stage her direction of movement is fixed by her stage-1 choice and, by Corollary 1, she has two actions: keep the object she currently holds, or choose the

[18] Indeed, let R be any such set. Then $R_0 \subseteq R$, and if $\Phi^k(R_0) \subseteq R$ then $\Phi^{k+1}(R_0) \subseteq \Phi(R) = R$ by monotonicity, so $\Phi^k(R_0) \subseteq R$ for every k. Since R^* is one of these iterates, $R^* \subseteq R$.

adjacent available object in her direction. Truthful behavior selects, at each decision, the action that moves toward the most preferred object she can still reach. At stage 1, keep the endowment if the endowment is her peak and otherwise choose the adjacent object on the side of the peak. At later stages, by Lemma 2, choose the adjacent available object if she prefers it to the object she currently holds, and keep the object she holds otherwise.

Consider any deviating strategy and any earliest point of departure I_i. Let o be the object i holds at I_i. Note that since i observes what she holds, o is the same at every history in I_i. Let a be the truthful action at I_i and $a' \neq a$ the deviating action. We establish two observations:

(L1) every outcome of truthful behavior from I_i is weakly preferred to o;

(L2) every outcome that can follow the deviating action a' is weakly worse than o.

Together these imply that the worst outcome following the truthful action is weakly preferred to the best outcome following the deviation, which is the obvious-dominance requirement of Appendix B.

Proof of (L1). If a is to keep o, truthful behavior assigns o and the claim is immediate. Suppose a chooses the adjacent available object. Under truthful behavior i's holding changes only through exchanges she herself chooses, and by Lemma 2 she chooses the adjacent available object only when she strictly prefers it to what she currently holds. Every exchange under truthful behavior therefore strictly improves what she holds, and the procedure ends for her either when she decides to keep or when her assignment becomes final, in both cases assigning what she holds at that moment. By transitivity her final assignment is weakly preferred to o.

Proof of (L2). Suppose i moves in some direction d at I_i, and note which objects she can then obtain. Her direction is fixed thereafter, so her holding can change only through exchanges into adjacent available objects in direction d. Every object she can subsequently receive is thus either o or an object beyond o in direction d. Since $a' \neq a$ there are three cases.

Case 1: a' keeps o while a moves. The deviation assigns o, so the outcome following a' is o.

Case 2: a' moves while a keeps o. Because keeping is truthful, o is the most preferred object i can still reach, so every available object in the direction chosen by a' is strictly worse than o. By the observation at the start of this step, every outcome following a' is o or such an object.

Case 3: a' and a move in opposite directions. This can happen only at stage 1, where both directions are available. Since a is truthful, the peak lies in the direction chosen by a, so every object in the direction chosen by a' lies on the far side of o from the peak and is, by single-peakedness, strictly worse than o. Again every outcome following a' is o or such an object.

In all cases (L1) and (L2) hold, and both observations apply at every history in I_i and for every strategy profile of the other agents, as the definition in Appendix B requires. □

D Information Conveyed by the Implementations

This appendix asks how much an agent must be told for each gradual mechanism to run, and it establishes the two facts that Section 5 draws on. First, whatever an agent learns as play unfolds concerns her own choice and the objects that remain, never any particular other agent. Propositions 2 and 3 show that neither gradual mechanism ever tells her what another agent chose, and Remark 3 shows that the inferences still available to her are anonymous and present in both mechanisms in equal measure. Any explanation of a treatment difference that appeals to what subjects learn about one another is therefore ruled out by construction. Second, for the Gradual Crawler the answer is almost nothing at all. Proposition 4 shows that the Crawler can be run as a two-option procedure in which an agent sees only the object she currently holds and the nearest available object in her direction. This makes the analogy with the ascending clock auction, and it implies that the display of the full set of available objects in our experimental interface (Section 3) is a convenience for subjects rather than a requirement of the mechanism.[19]

The question differs from the one the literature on obvious strategy-proofness ordinarily asks, namely whether a rule admits some obviously strategy-proof implementation. For that question the revelation-principle results of Ashlagi and Gonczarowski (2018) and Mackenzie (2020) make it without loss of generality to restrict attention to perfect information, a reduction that adds information. We move in the opposite direction and ask how little suffices. Moving in that direction is not free. Coarsening an agent's information removes strategies, since a strategy must prescribe the same action at every history she cannot distinguish, so truthful behavior must be shown to remain admissible, and it makes obvious dominance harder to satisfy, since the worst truthful outcome and the best deviating outcome are each taken over more histories. Propositions 2 and 4 are in this sense stronger than obvious strategy-proofness of the Crawler, which Bade (2019) establishes with a perfect-information game form that questions agents one at a time (Remark 2).

Both definitions below describe an implementation by what each agent sees. We call the information the mechanism displays to agent i at a history h her *observation* at h, and say that an implementation is *generated* by a given notion of observation if, for each i, two histories at which i is called upon belong to the same information set if and only if they yield the same sequence of observations received and choices made up to and including that history. Recording the whole sequence, rather than only the current observation, is what makes an agent remember her own past, and it delivers perfect recall.

[19] The implementation we ran is the one of Definition 1.

Definition 1. For an agent i and a history h at which i is called upon, let $\iota_i(h) = \big(O^h,\ e^h(i)\big)$ denote agent i's observation at h: the set of available objects and the object she holds.

The stage index is absent from the observation, and the mechanism keeps it absent. An agent's information records only the observations she has received and the choices she has made, one entry per decision of hers. Two histories that yield the same such sequence are therefore indistinguishable to her even when they lie at different stages of the procedure, because she may wait through any number of stages between two of her decisions without being shown anything. This makes the information partition coarser, and by the observation above it makes Proposition 2 a stronger claim. It also aligns Definition 1 with Definition 2 below, in which the stage index likewise plays no role.

Proposition 2. The information partition of the Gradual Crawler is generated by ι_i, and truthful behavior is obviously dominant in it. No agent is ever told what any other agent chose, either within a stage or afterward.

Proof. By Corollary 1 the action set at any decision node of i is determined by her observation history. Her current observation gives $e^h(i)$ and O^h, and her direction of movement is fixed by her stage-1 choice, which is the first recorded choice in the history and determines the direction together with her endowment o_i. So the action set is the same at any two histories with the same observation history. Perfect recall holds because an information set consists of histories with the same sequence of observations and choices, so any two histories in it give i the same experience. Whether i is called upon at all is, by Lemma 1 and (1), also determined by her observation history. That is, she remains in the procedure if and only if the object she holds is still available, and she is asked to choose again if and only if her previous choice has left O^h or she has just exchanged. The latter is the case when her previous choice is the object she now holds. Hence grouping histories by the observation history yields a well-defined information partition for G, and the outcome function is unchanged.

By Lemma 2, truthful behavior as described in Proposition 1 prescribes the following. At stage 1, choose the endowment if the endowment is the peak and otherwise choose the adjacent object on the side of the peak, which depends only on the endowment and on P_i. At any later decision node, choose the adjacent available object $\mathrm{adj}(e^h(i), d_i, O^h)$ if she prefers it to $e^h(i)$, and $e^h(i)$ otherwise. Everything this prescription refers to is determined by her observation history. So truthful behavior is an admissible strategy in G. Note that this step is not immediate. A strategy must prescribe the same action at every history the agent cannot distinguish, so coarsening her information removes every strategy that conditions on a distinction she no longer sees. A strategy that is dominant when she observes more may therefore not even be available when she observes less.

It remains to check obvious dominance. Observations (L1) and (L2) in the proof of Proposition 1 (Appendix C) hold at every history in the relevant information set and for every strategy profile of the other agents. Their proof uses only three facts: that preferences are single-peaked, that the direction of movement is fixed once chosen, and that $e^h(i)$ is the same at every history in an information set. The first is an assumption on the domain. The second and third hold here because the direction and the object held are both determined by the observation history. Observations (L1) and (L2) therefore carry over unchanged, and they imply the obvious-dominance requirement of Appendix B.

Finally, each component of each observation in the sequence refers only to i or to the objects themselves, i.e., O^h is a set of objects, $e^h(i)$ is the object i holds, and the recorded choices are i's own. None of them records a choice made by any $j \neq i$, or the identity of any such agent. $\square$

An agent in the Gradual Crawler needs to know only three things: which objects are still available, which object she is holding, and what she herself has chosen so far. She is never told what any particular other agent did. Two features of the mechanism make this possible. Because agents choose simultaneously, being asked to choose conveys nothing about anyone else. And because whether an assignment becomes final depends only on the set of available objects (Lemma 1), the mechanism can proceed without attributing any outcome to a named agent.

The same holds for the Gradual TTC, and we state it because the comparison in Section 5 turns on the two gradual mechanisms being alike in this respect. There, an agent never holds different objects across stages, so the observation is simpler.

Proposition 3. The information partition of the Gradual TTC is generated by the observation $\tilde{\imath}_i(h) = O^h$, together with agent i's own past choices. Truthful behavior is weakly dominant in it, and no agent is ever told what any other agent chose, either within a stage or afterward.

Proof. At stage 1 the action set is O. At any later history at which i is called upon, her action set consists of the available objects immediately before and after her previous choice in index order, which is determined by O^h together with the last entry of her own recorded choices. So the action set is the same at any two histories with the same observation history, and perfect recall holds because an information set consists of histories with the same sequence of observations and choices. Whether i is called upon is likewise determined by that history. She is asked to choose again when her previous choice has left O^h. Indeed, a cycle containing i assigns her the object she chose and ends her participation, so at any non-terminal decision node of hers the departure of her previous choice from O^h means it was assigned to another agent. Hence grouping histories by the observation history yields a well-defined information partition, and the outcome function is unchanged. Truthful behavior refers only to P_i and to the set of available objects, so it is an admissible strategy, and it is weakly dominant by the argument of footnote 9, which uses only the evolution of the set of available objects. Finally, each component of each observation refers only to the objects themselves or to i's own choices, so none records a choice made by, or the identity of, any $j \neq i$. □

Table D.1: What each implementation of the Crawler conveys to an agent.

	Bade (2019) (Remark 2)	Gradual Crawler (Def. 1)	Two-option form (Def. 2)
Object currently held	yes	yes	yes
Nearest available object ahead	yes	yes	yes
Full set of available objects	yes	yes	no
Which agent took which object	yes	no	no
Being asked is itself informative	yes	no	no
Obviously strategy-proof	yes	yes	yes

Remark 2. Bade (2019) establishes obvious strategy-proofness of the Crawler using a perfect-information game form that questions agents one at a time. Agents are asked in ascending order of the object they currently occupy. Each may pass or choose an object no larger than the one she occupies, the occupant of the largest remaining object being required to choose, and a step ends when someone chooses. Relative to that game form, ours withholds the two items recorded in the last two rows of Table D.1. Being asked to move is itself informative there, since an agent is asked only after every agent holding a smaller-indexed object has passed, which tells her that each of those agents strictly prefers to move up, and each assignment is observed together with the identity of the agent who made it, so every later mover learns which agent took which object. Bade's dominant strategy and her obvious-dominance argument already refer only to the set of unassigned objects and the agent's position in it, never to who passed, so in this sense the logic of the Crawler was local from the start. What does not follow from that alone is the game form: questioning agents one at a time conveys information however the resulting histories are grouped into information sets. Simultaneous choices within a stage are what deliver the informational properties of Proposition 2, and they are also why we work with the imperfect-information definition of obvious strategy-proofness rather than its perfect-information special case.

Remark 3. Propositions 2 and 3 concern what agents are told, not what they can infer. Two inferences remain available. An agent whose choice is not realized learns that the holder of the object she named did not choose hers, and the evolution of O^h reveals that some objects have been assigned. Neither can be avoided in any implementation in which agents choose as the procedure unfolds: by Corollary 1 an agent's own action set already depends on O^h and on what she holds, so an implementation that withheld all such information could not present her with a well-defined choice. What matters for our purposes is that the residual inference is available in the two gradual mechanisms in equal measure and is in both cases anonymous: it concerns the holder of a named object, never an identified opponent's report. What can be reduced is how much of the set of available objects an agent is shown, and Proposition 4 shows how far that reduction can go.

Definition 2. For an agent i and a history h at which i is called upon, let $\hat{\iota}_i(h) = \big(e^h(i),\ d_i,\ \mathrm{adj}(e^h(i), d_i, O^h)\big)$ denote agent i's observation at h: the object she holds, her direction of movement, and the nearest available object in that direction, with the last two entries omitted at stage 1. At stage 1 the direction is not yet determined, since it is fixed by the choice i is about to make, and the adjacent object is undefined along with it; nothing is lost, because every object is still available at stage 1. So the objects adjacent to i's endowment are determined by that endowment alone.

Proposition 4. Let $\widehat{G}$ be the game form obtained from the Gradual Crawler by replacing each agent's information partition with the one generated by $\hat{\iota}_i$. Then $\widehat{G}$ is a well-defined extensive game form with perfect recall, it implements the Crawler, and truthful behavior is obviously dominant in it. Hence the Crawler admits an obviously strategy-proof implementation in which each agent observes only the object she currently holds and the nearest available object in her direction of movement.

Proof. By Corollary 1 the action set at a decision node of agent i at a stage $k \geq 2$ is $\{e^h(i), \mathrm{adj}(e^h(i), d_i, O^h)\}$, which is the current observation itself, and at stage 1 it is determined by her endowment, which is her first observation. In either case the action set is determined by her observation history. Perfect recall holds because an information set consists of histories with the same sequence of observations and choices. Grouping histories by the $\hat{\iota}_i$-sequence is coarser than grouping them by the ι_i-sequence, i.e., $e^h(i)$ is a component of $\iota_i(h)$, the direction is determined by the recorded choices together with the endowment, and the adjacent object is determined by O^h and these, so histories with the same ι_i-sequence and choices have the same $\hat{\iota}_i$-sequence. Changing an agent's information partition leaves the histories, actions, and outcome function of the game form unchanged.

By Lemma 2 applied with $O' = O^h$, $o = e^h(i)$, and $d = d_i$, truthful behavior at any decision node of i at a stage $k \geq 2$ prescribes as follows. Choose $\mathrm{adj}(e^h(i), d_i, O^h)$ if she prefers it to $e^h(i)$, and keep $e^h(i)$ otherwise. This refers to h only through her current observation $\hat{\iota}_i(h)$. At stage 1 it refers only to her endowment and P_i. Truthful behavior is therefore an admissible strategy in $\widehat{G}$, and since $\widehat{G}$ has the same histories and outcome function as G it produces the same allocation. Hence $\widehat{G}$ implements the Crawler.

As in the proof of Proposition 2, observations (L1) and (L2) of Proposition 1 (Appendix C) hold at every history at which i holds a given object and has a given direction of movement, and for every strategy profile of the other agents. Since the object held and the direction are determined by the $\hat{\iota}_i$-sequence, every information set of $\widehat{G}$ consists of such histories. Hence the obvious-dominance requirement holds at every earliest point of departure of $\widehat{G}$. $\square$

The Crawler can be run as a two-option procedure. An agent is shown the object she holds and the nearest available object in her direction, and asked whether to stop or continue. When no object remains ahead of her, her assignment is final and the procedure ends for her. She never sees the full set of available objects, and she never learns anything about the others. This is the sense in which the mechanism is the reallocation counterpart of an ascending clock. It also means that the display of available objects in our experimental interface (Section 3) is a design choice rather than a requirement of the mechanism.

Remark 4. For $n \geq 4$ the information in Definition 2 is strictly less than that in Definition 1. Take $n = 5$, consider agent 1 with endowment o_1, and compare two profiles of stage-1 choices in each of which agent 1 chooses o_2, agent 2 chooses o_2, and agent 5 chooses o_5.

(a) Agent 3 chooses o_2 and agent 4 chooses o_3. No two agents choose each other's objects, so no exchange occurs; agents 2 and 5 are assigned, so the objects available at stage 2 are $\{o_1, o_3, o_4\}$.

(b) Agent 3 chooses o_4 and agent 4 chooses o_3. Agents 3 and 4 exchange; agent 3 then holds o_4, moving in the direction of larger indices, and o_4 is the largest available object, so her assignment is final. The objects available at stage 2 are therefore $\{o_1, o_3\}$.

In both cases agent 1's sequence of observations and choices under Definition 2 is the same. At stage 1 she observes her endowment o_1 and chooses o_2. At stage 2, her choice having been assigned to someone else. So she observes that she holds o_1, moves toward larger indices, and has adjacent available object o_3. Under Definition 1 her stage-2 observations differ, since the sets of available objects differ. The two histories therefore lie in the same information set of $\widehat{G}$ but in different information sets of the Gradual Crawler.

Remark 5. Proposition 4 is minimal in the following sense. By Corollary 1, the object an agent holds and the adjacent available object together are her action set, so no implementation can withhold both. An agent must be told enough to know what she is choosing between. Definition 2 tells her exactly that and nothing more. Whether some intermediate notion of observation, incomparable to $\hat{\iota}_i$, also supports obvious dominance we leave open. What Proposition 4 shows is that nothing beyond what the action set already reveals is ever needed.

Remark 6. Our Gradual TTC and Gradual Crawler are not pick-an-object mechanisms in the sense of Bó and Hakimov (2024). Under their definition an agent's menu can only shrink over time, i.e., it never contains an object absent from an earlier menu. In our gradual mechanisms an object unavailable at an earlier stage may become available to an agent later.

E Balance Checks

Table E.1: Demographic Descriptive Statistics

	Age (Years)		Gender (Female)	
Treatment	Mean	SD	Observations	Percentage (%)
Gradual Crawler	23.26	2.55	23	46.00
Gradual TTC	22.60	2.29	24	48.00
Static Crawler	22.94	2.80	17	34.00
Static TTC	21.58	2.17	24	48.00

Table E.2: Student's t-Tests for Age Balance

Treatment	(1)	(2)	(3)	(4)
(1) Gradual Crawler	–	0.176	0.551	$< 0.001^{***}$
(2) Gradual TTC	0.176	–	0.507	0.024^{**}
(3) Static Crawler	0.551	0.507	–	0.008^{***}
(4) Static TTC	$< 0.001^{***}$	0.024^{**}	0.008^{***}	–

Table E.3: Mann-Whitney U Tests for Gender Balance

Treatment	(1)	(2)	(3)	(4)
(1) Gradual Crawler	–	0.629	0.362	0.845
(2) Gradual TTC	0.629	–	0.172	0.772
(3) Static Crawler	0.362	0.172	–	0.269
(4) Static TTC	0.845	0.772	0.269	–

F Tests for Learning

Table F.1: Learning Effects for Average Efficiency

Treatment	Pareto 1–10 (%)	Pareto 11–20 (%)	Pareto p-val	Max 1–10 (%)	Max 11–20 (%)	Max p-val
Gradual Crawler	90.60	99.40	$< 0.001^{***}$	89.78	97.13	$< 0.001^{***}$
Gradual TTC	81.54	82.20	0.614	80.41	81.35	0.555
Static Crawler	81.61	83.01	0.443	80.58	82.31	0.371
Static TTC	86.38	88.23	0.230	85.83	87.43	0.252

Note: *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively. Non-parametric p-values are generated using two-sided Wilcoxon Rank-Sum tests at the group-round level.

Table F.2: Learning Effects for Average Efficiency (from period 4)

Treatment	Pareto 4–10 (%)	Pareto 11–20 (%)	Pareto p-val	Max 4–10 (%)	Max 11–20 (%)	Max p-val
Gradual Crawler	94.69	99.40	0.006^{***}	93.90	97.13	0.043^{**}
Gradual TTC	83.61	82.20	0.582	82.72	81.35	0.557
Static Crawler	82.80	83.01	0.859	81.95	82.31	0.787
Static TTC	88.72	88.23	0.932	88.01	87.43	0.878

Note: *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively. Non-parametric p-values are generated using two-sided Wilcoxon Rank-Sum tests at the group-round level.

G Pairwise Tests

Table G.1: Treatment Comparisons: Pairwise Mann-Whitney U Tests (Pareto Efficiency, Uncorrected)

Treatment	(1)	(2)	(3)	(4)
(1) Gradual Crawler	–	$< 0.001^{***}$	$< 0.001^{***}$	$< 0.001^{***}$
(2) Gradual TTC	$< 0.001^{***}$	–	0.557	$< 0.001^{***}$
(3) Static Crawler	$< 0.001^{***}$	0.557	–	$< 0.001^{***}$
(4) Static TTC	$< 0.001^{***}$	$< 0.001^{***}$	$< 0.001^{***}$	–

Note: *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table G.2: Treatment Comparisons: Pairwise Mann-Whitney U Tests (Pareto Efficiency, Bonferroni Corrected)

Treatment	(1)	(2)	(3)	(4)
(1) Gradual Crawler	–	$< 0.001^{***}$	$< 0.001^{***}$	$< 0.001^{***}$
(2) Gradual TTC	$< 0.001^{***}$	–	1.000	$< 0.001^{***}$
(3) Static Crawler	$< 0.001^{***}$	1.000	–	0.002^{***}
(4) Static TTC	$< 0.001^{***}$	$< 0.001^{***}$	0.002^{***}	–

Note: *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively. p-values are adjusted using the Bonferroni correction for 6 pairwise comparisons.

Table G.3: Treatment Comparisons: Pairwise Mann-Whitney U Tests (Max Pareto Efficiency, Uncorrected)

Treatment	(1)	(2)	(3)	(4)
(1) Gradual Crawler	–	$< 0.001^{***}$	$< 0.001^{***}$	$< 0.001^{***}$
(2) Gradual TTC	$< 0.001^{***}$	–	0.587	$< 0.001^{***}$
(3) Static Crawler	$< 0.001^{***}$	0.587	–	$< 0.001^{***}$
(4) Static TTC	$< 0.001^{***}$	$< 0.001^{***}$	$< 0.001^{***}$	–

Note: *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table G.4: Treatment Comparisons: Pairwise Mann-Whitney U Tests (Max Pareto Efficiency, Bonferroni Corrected)

Treatment	(1)	(2)	(3)	(4)
(1) Gradual Crawler	–	$< 0.001^{***}$	$< 0.001^{***}$	$< 0.001^{***}$
(2) Gradual TTC	$< 0.001^{***}$	–	1.000	$< 0.001^{***}$
(3) Static Crawler	$< 0.001^{***}$	1.000	–	0.001^{***}
(4) Static TTC	$< 0.001^{***}$	$< 0.001^{***}$	0.001^{***}	–

Note: *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively. p-values are adjusted using the Bonferroni correction for 6 pairwise comparisons.

Table G.5: Pairwise Mann-Whitney U Matrix: First-Move Truthful (Raw p-values)

Treatment	(1)	(2)	(3)	(4)
(1) Gradual Crawler	–	$< 0.001^{***}$	$< 0.001^{***}$	$< 0.001^{***}$
(2) Gradual TTC	$< 0.001^{***}$	–	0.3761	$< 0.001^{***}$
(3) Static Crawler	$< 0.001^{***}$	0.3761	–	$< 0.001^{***}$
(4) Static TTC	$< 0.001^{***}$	$< 0.001^{***}$	$< 0.001^{***}$	–

Notes: Table indices: (1) Gradual Crawler, (2) Gradual TTC, (3) Static Crawler, (4) Static TTC. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table G.6: Pairwise Mann-Whitney U Matrix: First-Move Truthful (Bonferroni Corrected p-values)

Treatment	(1)	(2)	(3)	(4)
(1) Gradual Crawler	–	$< 0.001^{***}$	$< 0.001^{***}$	$< 0.001^{***}$
(2) Gradual TTC	$< 0.001^{***}$	–	1.0000	$< 0.001^{***}$
(3) Static Crawler	$< 0.001^{***}$	1.0000	–	0.0017^{***}
(4) Static TTC	$< 0.001^{***}$	$< 0.001^{***}$	0.0017^{***}	–

Notes: Table indices: (1) Gradual Crawler, (2) Gradual TTC, (3) Static Crawler, (4) Static TTC. p-values adjusted for 6 multiple comparisons.
$^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table G.7: Pairwise Mann-Whitney U Matrix: Strict Truthful (Raw p-values)

Treatment	(1)	(2)	(3)	(4)
(1) Gradual Crawler	–	$< 0.001^{***}$	$< 0.001^{***}$	0.3633
(2) Gradual TTC	$< 0.001^{***}$	–	0.3411	$< 0.001^{***}$
(3) Static Crawler	$< 0.001^{***}$	0.3411	–	$< 0.001^{***}$
(4) Static TTC	0.3633	$< 0.001^{***}$	$< 0.001^{***}$	–

Notes: Table indices: (1) Gradual Crawler, (2) Gradual TTC, (3) Static Crawler, (4) Static TTC.
$^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table G.8: Pairwise Mann-Whitney U Matrix: Strict Truthful (Bonferroni Corrected p-values)

Treatment	(1)	(2)	(3)	(4)
(1) Gradual Crawler	–	0.0057^{***}	0.0013^{***}	1.0000
(2) Gradual TTC	0.0057^{***}	–	1.0000	0.0032^{***}
(3) Static Crawler	0.0013^{***}	1.0000	–	$< 0.001^{***}$
(4) Static TTC	1.0000	0.0032^{***}	$< 0.001^{***}$	–

Notes: Table indices: (1) Gradual Crawler, (2) Gradual TTC, (3) Static Crawler, (4) Static TTC. p-values adjusted for 6 multiple comparisons.
$^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table G.9: Pairwise Mann-Whitney U Matrix: IR Violations (Raw p-values)

Treatment	(1)	(2)	(3)	(4)
(1) Gradual Crawler	–	0.3140	1.0000	0.1818
(2) Gradual TTC	0.3140	–	0.3140	0.0269^{**}
(3) Static Crawler	1.0000	0.3140	–	0.1818
(4) Static TTC	0.1818	0.0269^{**}	0.1818	–

Notes: Table indices: (1) Gradual Crawler, (2) Gradual TTC, (3) Static Crawler, (4) Static TTC.
$^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table G.10: Pairwise Mann-Whitney U Matrix: IR Violations (Bonferroni Corrected p-values)

Treatment	(1)	(2)	(3)	(4)
(1) Gradual Crawler	–	1.0000	1.0000	1.0000
(2) Gradual TTC	1.0000	–	1.0000	0.1615
(3) Static Crawler	1.0000	1.0000	–	1.0000
(4) Static TTC	1.0000	0.1615	1.0000	–

Notes: Table indices: (1) Gradual Crawler, (2) Gradual TTC, (3) Static Crawler, (4) Static TTC. p-values adjusted for 6 multiple comparisons.
$^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

H Regression Results

Table H.1: Treatment Effects on Efficiency and Choice Behavior

	Efficiency (1)	Maximum Efficiency (2)	First-Move Truth-telling (3)	Strict Truth-telling (4)	IR Violations (5)
Constant (Gradual Crawler)	0.935***	0.925***	0.942***	0.587***	0.003*
	(0.002)	(0.002)	(0.007)	(0.025)	(0.002)
Gradual TTC	-0.128***	-0.127***	-0.419***	-0.213***	-0.002
	(0.028)	(0.028)	(0.021)	(0.033)	(0.002)
Static Crawler	-0.123*	-0.122*	-0.391***	-0.093***	0.000
	(0.054)	(0.053)	(0.023)	(0.034)	(0.002)
Static TTC	-0.071	-0.070	-0.249***	0.074**	0.004
	(0.045)	(0.044)	(0.026)	(0.036)	(0.004)
Observations	800	800	4000	4000	4000
Clusters	8	8	200	200	200

Notes: OLS Linear Probability Model regressions. Robust standard errors are clustered at the session level for efficiency measure (1-2) and at the individual level for thruth-telling and IR (3-5), reported in parentheses. Significance levels: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Online Appendix for "Top Trading Cycles vs. the Crawler: The Role of Obvious Strategy-Proofness"

A Strategic Complexity

All four of our implementations are strategy-proof, and only the Gradual Crawler is obviously strategy-proof. Obvious strategy-proofness therefore separates the Gradual Crawler from the other three implementations but delivers no prediction about their relative behavioral performance. The measure of strategic complexity of Nagel and Saitto (2026) ranks any two strategy-proof mechanisms by how much contingent reasoning an agent must do to recognize that truthful behavior is optimal. This online appendix states the measure precisely and computes it for each of the four mechanisms. Table 4 in the main text collects the results.

In either static mechanism an agent chooses among the 2^{n-1} single-peaked rankings, and in the worst case only one can be discarded: the report that guarantees her the endowment. Every other report can reach her most preferred object but can also leave her with the endowment or worse, so no two of them can be compared for free, and complexity is $2^{n-1} - 1$, or 15 with five agents. In the Gradual TTC the first choice is among the n objects, and in the worst case nothing can be discarded, not even the choice of her endowment.[1] Thus complexity is n, or 5. The Gradual Crawler requires a different treatment, because for $n \geq 4$ it is not an *essential frame* (Remark OA.2): there are decisions of the same agent between which nothing payoff-relevant is revealed, so a plan can bundle several of

[1] Although that choice guarantees the endowment, every other choice leaves open outcomes both better and worse than the endowment, so no choice's guarantee is met by another's worst case.

her binary choices. This can happen in two ways. The object she has chosen may be assigned to someone else while, for some preference relation consistent with what she has observed, the set of payoffs she can still attain is unchanged. And when another agent's earlier choice names the object she currently holds, her next exchange is certain if she continues, so her truthful play becomes deterministic and nothing can ever be revealed to her again. Computed at the level of plans, the complexity of the Gradual Crawler is 2 for $n = 3$, 3 for $n = 4$, and 5 for $n = 5$; Lemmas OA.6 and OA.7 contain the formal analysis, including why the bundled plans cannot be discarded.

At $n = 5$ the measure thus separates the two static mechanisms from the two gradual ones. However, it assigns the same complexity to the Gradual TTC and the Gradual Crawler, and it ranks both gradual mechanisms as strictly simpler than both static ones. Neither feature matches our data, in which the two gradual mechanisms are the best and the worst performers.

A.1 The Measure

We restate the definitions of Nagel and Saitto (2026) in the notation of this paper. In our environment, an agent's private information is her preference relation. Thus, what Nagel and Saitto (2026) call a type is here a single-peaked preference relation P_i, and we use only that term. Fix a strategy-proof mechanism, viewed as an extensive game form together with a profile of dominant strategies. Throughout, the dominant strategies are the truthful ones, and all statements about what can happen from an information set onward are conditional on that information set being reached.[2]

We say that P_i is *consistent* with an information set I of agent i if an agent with preferences P_i reaches I under her truthful strategy for some behavior of the others. An object is *attainable* for P_i at I if she receives it when she follows her truthful strategy from I onward and every other agent follows her own truthful

[2]This is consistent with Nagel and Saitto (2026), whose definitions evaluate outcomes conditional on reaching the information set. They also note that their results are unchanged if the worst case below is taken only over information sets reached under the truthful strategy profile. In our four mechanisms every information set is so reached, by the minimality arguments in the proof of Proposition OA.1 and the enumeration behind Proposition OA.2, so the distinction never arises.

strategy, for some profile of the others' preference relations consistent with I. Finally, receiving an object is *outside her control* at I if whether she receives it does not depend on her own behavior.[3]

Nagel and Saitto (2026) assume that an agent does not choose between two continuation behaviors that first differ only after she will have learned something new about the payoffs she can still influence. Formally, payoff-relevant information is *revealed* between two information sets $I \prec I'$ of agent i if both are reached under the truthful strategy profile and, for each preference relation P_i consistent with I', some object attainable for P_i at I can no longer be received at I' when she follows her truthful strategy, whatever the behavior of the others consistent with I'. If payoff-relevant information is revealed between I and some information set, it is also revealed between I and every information set beyond it, since the consistent preference relations and the objects that can still be received only shrink along the tree. It therefore suffices to check the condition for consecutive information sets. Two continuation behaviors at I belong to the same *continuation plan* if and only if they agree at I and at every subsequent information set of hers between which and I no payoff-relevant information is revealed. The agent's problem at I is to choose a plan.[4]

Which objects can follow a plan at I is determined by the action the plan takes at I: they are the objects the agent can receive after taking that action, over all behaviors of the others consistent with I and over all of her own continuations, excluding objects whose receipt is outside her control. An object is a *menu option* at I if every continuation behavior of hers under which it can be received yields it with certainty, whatever the others do. In other words, a menu option is an object the agent can guarantee to herself, and one she can never receive without having guaranteed it. A plan D' is *evidently never better* than a plan $D'' \neq D'$ for P_i if the best object that can follow D' is weakly worse, according to P_i, than the worst object that can follow D''. In this comparison, an object that can follow D'' is not counted if some behavior in D'' secures a menu option that P_i ranks above it: ending in such an object would require the agent to pass up something better that

[3]In our mechanisms, receiving an object is outside an agent's control at an information set only if she cannot receive it at all from that information set; see the second convention in the next subsection.

[4]The choices on which a plan is silent are deferred until the information arrives.

she could simply have taken. Recognizing that D' can be discarded then requires no reasoning about what the others will do or about how the agent herself will continue. A set of plans can be *evidently reduced* by P_i if it contains two plans $D' \neq D''$ such that D' does not contain the continuation of her truthful strategy and D' is evidently never better than D''.

Consider an information set I, a preference relation P_i consistent with I, and a set of continuation plans at I that contains the truthful plan of P_i. We call the set *irreducible* if it cannot be evidently reduced by P_i. The *strategic complexity* of the mechanism is the largest cardinality of an irreducible set, over all agents, information sets, and preference relations consistent with the information set.

A strategy-proof mechanism is an *essential frame* if (i) payoff-relevant information is revealed between any two consecutive information sets of the same agent, and (ii) its game tree is minimal, in the sense that complexity cannot be weakly lowered by deleting histories that are never reached under the truthful strategy profile or deleting information sets with a single action. In an essential frame, every continuation plan prescribes a single action, so plans coincide with the actions available at the information set, which Nagel and Saitto (2026) call messages. Menu options then play no role. Nagel and Saitto (2026) show that in an essential frame, an action after which a menu option can be received leads to that menu option with certainty, so no other object can follow such an action and there is nothing to discard. The comparison of plans therefore simplifies to a comparison of actions: a' is evidently never better than a'' for P_i if the best object that can follow a' is weakly worse, according to P_i, than the worst object that can follow a''. Proposition 1 of Nagel and Saitto (2026) then states that the strategic complexity of an essential frame equals the largest cardinality of an irreducible set of actions at any information set. The static mechanisms and the Gradual TTC are essential frames (Proposition OA.1). The Gradual Crawler is not, for $n \geq 4$, and its complexity is computed directly (Proposition OA.2 and Remark OA.2).

A.2 Setup, Notation, and Conventions

Throughout we work in the environment of Section 2.1 of the main text, generalized to $n \geq 3$ agents with objects $o_1, \ldots, o_n$ on the line and single-peaked preferences.

Agent i's endowment is o_i and p_i is the peak index of P_i. For a static mechanism and a report m, i.e., a single-peaked ranking submitted by the agent, we denote by $\mathcal{O}_i(m)$ the set of objects agent i can receive after reporting m, over all reports of the others. We write P^m and R^m for the strict and weak rankings given by m. For a gradual mechanism, an information set I of agent i, and an action m available at I, we denote by $\mathcal{O}_i(I, m)$ the set of objects i can receive after choosing m at I, over all continuations of her own behavior and all behavior of the others. At stage 1 of the Gradual TTC an action is the choice of an object. When agent i chooses an object o_j with $j \neq i$, say that o_j lies to the *right* (*left*) of her endowment if $j > i$ ($j < i$), and define

$$S_i(o_j) = \begin{cases} \{o_i, o_{i+1}, \ldots, o_n\} & \text{if } j > i, \\ \{o_1, \ldots, o_{i-1}, o_i\} & \text{if } j < i, \end{cases}$$

the set of objects from her endowment to the end of the line on the side of o_j.

Three conventions adapt the definitions of Nagel and Saitto (2026) to our setting. First, we suppress information sets at which only one action is available. Whenever an agent in the Gradual TTC has only one object to choose from because her previous choice was extreme in the set of available objects, the choice is made automatically. This changes neither behavior nor outcomes and is required by the minimality condition of an essential frame. We call the remaining information sets, at which at least two actions are available, *active*. In the Gradual Crawler this convention has no effect. By Lemma 1 and Corollary 1 of the main text, an agent who holds the extreme available object in her direction receives a final assignment and leaves. So every agent still in the procedure has at least two actions.

Second, the definitions of Section A.1 disregard two kinds of objects: those whose receipt is outside the agent's control, and, on one side of each comparison, those ranked below a menu option. The first has no effect in our mechanisms. At every active information set at which the agent can receive at least two different objects, the receipt of every such object depends on her own behavior. In a static mechanism, a report ranking her endowment first secures it, while, by Lemma OA.1 or OA.2, under some other report together with suitable reports of the others she receives a different object. In the Gradual Crawler, choosing the object she holds secures it immediately, so that, fixing the others' choices along a play that reaches any other receivable object, her own behavior decides between the two, and the receipt of neither is outside her control. In the Gradual TTC, choosing her endowment at stage 1 secures it, while after any other stage-1 choice she can receive a different object by Lemma OA.3. For the later active information sets, the proof of Proposition OA.1 shows that for some behavior of the others she receives whichever of her two available objects she chooses. Hence the objects outside an agent's control at such an information set are exactly the ones she cannot receive at all. The objects that can follow a report m are therefore exactly $\mathcal{O}_i(m)$, and the objects that can follow a plan taking action m at I are exactly $\mathcal{O}_i(I, m)$.[5] The second removal is vacuous for essential frames, as noted in Section A.1, and therefore plays no role in Proposition OA.1. It is essential for the Gradual Crawler, where it is handled in Lemma OA.7 and Remark OA.1.

Third, the measure is defined relative to a designated profile of dominant strategies. For the static mechanisms this is truthful reporting. For the gradual mechanisms it is the truthful behavior described in Section 2 of the main text, which is dominant in the Gradual TTC and obviously dominant in the Gradual Crawler by Proposition 1 of the main text.

A.3 What Agents Can Receive

The lemmas in this subsection characterize the sets $\mathcal{O}_i(m)$ and $\mathcal{O}_i(I, m)$ defined in the previous subsection. By the second convention, these are the sets of objects over which the comparisons of Nagel and Saitto (2026) are taken.

Lemma OA.1. *In the Static TTC on the single-peaked domain, fix agent i and a report m. Then $\mathcal{O}_i(m) = \{o \in O : o\ R^m\ o_i\}$.*

Proof. ($\subseteq$) TTC assigns i the best object according to R^m among those remaining when she forms a cycle. Since o_i is removed only in a cycle containing i, the set of remaining objects always contains o_i, so her assignment is weakly better than o_i according to R^m.

[5] At an active information set at which the agent can receive only one object, its receipt does not depend on her behavior, every comparison holds trivially, and such information sets contribute at most one plan to the complexity.

($\supseteq$) Let $o \in O$ be such that $o \; R^m \; o_i$, and let T be the set of objects ranked strictly above o by m. Let each owner of an object in T report a ranking with her own endowment first, and let every other agent $j \neq i$ report a ranking with peak o_i. Suppose that $T \neq \emptyset$. In round 1 the owners of T leave with their endowments, and no other cycle forms, since every remaining agent other than i points at agent i, while i points at the owner of an object in T. In round 2, if $o \neq o_i$, agent i points at the owner of o, who points at i, so the two form a cycle and o is assigned to i. If $o = o_i$, agent i points at herself and leaves with o_i. If $T = \emptyset$, the same cycles form already in round 1. □

Lemma OA.2. *In the Static Crawler on the single-peaked domain, fix agent i and a report m. Then $\mathcal{O}_i(m) = \{o \in O : o \; R^m \; o_i\}$.*

Proof. ($\subseteq$) The Crawler is individually rational (Bade, 2019). So i's assignment is weakly better than her endowment according to R^m.

($\supseteq$) Let $o_k \in O$ be such that $o_k \; R^m \; o_i$. If $k = i$, let every agent $j \neq i$ report a ranking with peak o_j. Then every agent $j \neq i$ most prefers her own endowment among the remaining objects, keeps it, and leaves in step 1. Thus i, facing only o_i, keeps it and receives o_i. Suppose $k \neq i$, and without loss of generality $k > i$. Since $o_k \; P^m \; o_i$ with $k > i$, the peak of m lies strictly to the right of o_i. Let every agent $j \notin \{i, k\}$ report a ranking with peak o_j, and let agent k report a ranking with peak o_i. In step 1 every $j \notin \{i, k\}$ most prefers her own endowment among the remaining objects, so she keeps it and leaves. Neither i nor k leaves in step 1. The objects o_i and o_k are now adjacent in the set of remaining objects, and each of the two agents ranks the other's current endowment above her own, so they exchange in step 2. Agent k then holds her peak o_i and leaves, and i holds o_k, the only remaining object, and receives it. □

Lemma OA.3. *In the Gradual TTC, fix agent i and her stage-1 information set I_i^1. Then $\mathcal{O}_i(I_i^1, o_i) = \{o_i\}$ and, for each $j \neq i$, $\mathcal{O}_i(I_i^1, o_j) = S_i(o_j)$.*

Proof. Choosing o_i forms a self-cycle, that is, a cycle in which she points at herself, so i receives o_i at once. Now fix $j \neq i$ and without loss of generality $j > i$. In the constructions below, every agent whose behavior is not specified chooses her own endowment at stage 1 and leaves with it. This removes only her endowment and never interferes with the cycles described.

$\mathcal{O}_i(I_i^1, o_j) \subseteq S_i(o_j)$. Agent i's assignment is always an object she chooses. Object o_i is removed only through a cycle containing i, so it remains available while i is unassigned. Consequently her choices can move from the right side of o_i to the left side only by first selecting o_i. Once all objects strictly between her current choice and o_i have been removed, the adjacent available object with a smaller index is o_i, and choosing it assigns it to her. Hence every choice of i, and thus every possible assignment, lies in $S_i(o_j)$.

$S_i(o_j) \subseteq \mathcal{O}_i(I_i^1, o_j)$. Fix $o_k \in S_i(o_j)$. If $k = j$, let the owner of o_j choose o_i at stage 1. Then agents i and j form a cycle, and i receives o_j. If $k = i$, let the owners of $o_j, o_{j-1}, \ldots, o_{i+1}$ behave so that for $t = 0, 1, \ldots, j-i-1$ the owner of o_{j-t} forms a self-cycle at stage $t+1$. This is feasible because at stage 1 each such owner may choose o_j, and at each later stage the adjacent available object with a smaller index either is, or leads step by step to, that owner's endowment. Agent i then chooses again toward smaller indices at every stage. After $j - i$ removals her choice is o_i, which is assigned to her. Suppose either $i < k < j$ or $j < k \leq n$. We combine the two constructions. The owners of the objects between o_j and o_k, inclusive of o_j and exclusive of o_k, form self-cycles one stage after another, as in the previous construction, while i chooses again toward o_k. The owner of o_k chooses o_i from stage 1 onward, which is feasible because o_i remains available, so this owner is never forced to choose again. When i's choice reaches o_k, agents i and k form a cycle and i receives o_k. □

A.4 Two Lemmas on the Dynamics of the Gradual TTC

Lemma OA.4. *Consider any play of the Gradual TTC and any two stages $t < T$. Suppose object o is available at the beginning of stage T and some agent j chooses o at stage t. Then*

(i) agent j chooses o at every stage $t, t+1, \ldots, T$;

(ii) agent j belongs to no cycle at any stage $t, t+1, \ldots, T-1$;

(iii) agent j's endowment o_j is available at the beginning of stage T.

Proof. An object is removed only when assigned, and assignments are permanent. Since o is available at the beginning of stage T, it is available at every stage from t to T. By the rules of the mechanism, an agent whose previous choice is still available takes no action and keeps it, which gives (i). For (ii), a cycle containing j at a stage $t' \in [t, T-1]$ would assign j the object she chooses at t', which by (i) is o. This removes o before stage T, a contradiction. For (iii), suppose o_j were assigned at some stage t'. Then o_j would be the choice of some agent in a cycle at t', and that agent's successor in the cycle is the owner of o_j, namely j. Hence j would belong to a cycle at t', which is impossible for $t' < T$ by (ii). $\square$

Lemma OA.5. *Consider any play of the Gradual TTC, two stages $t < T$, and distinct agents $j_0, j_1, \ldots, j_r$ ($r \geq 1$) who are active at stage t such that j_m chooses $o_{j_{m+1}}$ at stage t for each $m = 0, \ldots, r-1$. If o_{j_r} is available at the beginning of stage T, then j_0 chooses o_{j_1} at every stage from t through T. In particular, j_0 makes no new choice at stage T.*

Proof. Applying part (iii) of Lemma OA.4 to j_{r-1}, whose stage-t choice o_{j_r} is available at the beginning of stage T, shows that $o_{j_{r-1}}$ is available at the beginning of stage T. Iterating for $j_{r-2}, \ldots, j_1$ shows that o_{j_1} is available at the beginning of stage T, and part (i) of Lemma OA.4 applied to j_0 gives the claim. $\square$

A.5 Complexity of the Static Mechanisms and Gradual TTC

Proposition OA.1. *In the environment of Section 2.1 of the main text, generalized to $n \geq 3$ agents, the Static TTC, the Static Crawler, and the Gradual TTC are essential frames, and their strategic complexities are $2^{n-1} - 1$, $2^{n-1} - 1$, and n respectively. Hence the Gradual TTC has the same complexity as the two static mechanisms at $n = 3$ and is strictly simpler than either if and only if $n \geq 4$. For $n = 5$ the complexities are 15, 15, and 5.*

Proof for the static mechanisms. In either static mechanism each agent has a single information set, so the information-revelation condition holds trivially. Every single-peaked report is the truthful report of a unique preference relation. So the game tree is minimal. Both are therefore essential frames in which an agent's actions are the 2^{n-1} single-peaked reports, and Proposition 1 of Nagel and Saitto (2026) applies. By Lemmas OA.1 and OA.2, $\mathcal{O}_i(m)$ is the same in both mechanisms, so one argument covers both.

We first bound complexity from above. Fix a preference relation P_i with endowment o_i. By Lemma OA.1, the objects that can follow the truthful report are exactly those weakly better than o_i according to P_i. So the worst object that can follow the truthful report is o_i. Moreover, every report ranking o_i first assigns o_i to i regardless of the other agents' reports.[6] Suppose that the peak of P_i differs from o_i. Then no report ranking o_i first is truthful, and every such report is evidently never better than the truthful report, so no irreducible set contains a report that ranks o_i first. Since at least one report ranks o_i first, complexity is at most $2^{n-1}-1$. If instead the peak of P_i equals o_i, the truthful report itself ranks o_i first, and every other report is evidently never better than it, so the irreducible set is a singleton.

For the lower bound, consider agent 1 and the preference relation P_1 given by $o_2\ P_1\ o_1\ P_1\ o_3\ P_1\ \cdots\ P_1\ o_n$. Because o_1 is an endpoint of the line, exactly one single-peaked report ranks o_1 first. Every other report m has its peak weakly to the right of o_2 and hence ranks o_2 above o_1. So $o_2 \in \mathcal{O}_1(m)$ and the best object that can follow m is the peak o_2. On the other hand, the worst object that can follow any report is weakly worse than o_1 according to P_1, since the endowment can follow every report. No report among these $2^{n-1}-1$ is therefore evidently never better than another, and the set of all of them contains the truthful report and is irreducible. $\square$

Proof that the Gradual TTC is an essential frame. Fix an agent i and two consecutive active information sets $I'_i \prec I''_i$. Fix a history $h' \in I'_i$, and let T be the stage at which i moves at h'. The argument below applies to each history in I'_i in the same way.

Reaching I''_i requires having made the choice recorded in its observation history. So each preference relation consistent with I''_i prescribes the same action at I'_i. Let o' be the object chosen at I'_i. Let $o'' \neq o'$ be another action available at I'_i, which exists since I'_i is active. Both o' and o'' are available at the beginning of stage T. Moreover:

(P1) $o' \neq o_i$. Choosing one's own endowment forms a self-cycle and ends the

[6] Under TTC she forms a self-cycle in round 1, and under the Crawler she most prefers her endowment among the remaining objects and leaves with it in step 1.

agent's participation. So no preference relation whose truthful strategy chooses o_i at I_i' reaches I_i''.

(P2) Agent i chooses neither o' nor o'' at any stage before T. Otherwise, since both are available at the beginning of stage T, part (i) of Lemma OA.4 would imply that she keeps that choice through stage T and makes no new choice there, contradicting that I_i' is active.

Agent i chooses again only once the object she chose has been assigned to another agent. Hence reaching I_i'' requires that o' has been assigned, and o' is unavailable from I_i'' onward. Since an agent is only ever assigned an available object, o' is not attainable at I_i''.

It remains to show that o' is attainable at I_i' for every preference relation consistent with I_i''. Fix such a preference relation P_i, a history $h'' \in I_i''$ with $h' \prec h''$, and a profile P_{-i} of preference relations of the others that generates h'' under truthful behavior.[7] Let w be the owner of o' and, if $o'' \neq o_i$, let u be the owner of o''. Note that $w \neq i$ by (P1) and $u \neq w$. Define $\tilde{P}_{-i}$ from P_{-i} by replacing the preference relations of w and of u, when the latter is defined, with single-peaked preference relations whose peak is o_i, leaving all others unchanged. We call these agents, i.e., agents w and u, *redirected*. Under truthful behavior a redirected agent chooses o_i at stage 1 and keeps that choice as long as o_i is available, that is, as long as i is unassigned.

Claim OA.1. *Under $(P_i, \tilde{P}_{-i})$, for every stage $t \leq T$, the set of available objects, the set of active agents, and the choices of all non-redirected agents coincide with those under (P_i, P_{-i}). Hence no cycle containing agent i forms at any stage $t < T$.*

Proof of Claim OA.1. The proof is by induction on t. At $t = 1$ all objects are available and all agents are active under both profiles. Every non-redirected agent has the same preference relation and hence chooses the same object, her peak. For the induction step, suppose the statement holds up to stage $t \leq T$. In any cycle, the agent who chooses o_i is followed by the owner of o_i, namely agent i. Since a redirected agent chooses o_i, any cycle containing a redirected agent therefore

[7]Such a profile exists by the minimality argument at the end of this proof, which shows that every action at every active information set is truthful for some consistent preference relation, so that every history is on the path of truthful play for some preference profile.

contains i. Consequently a cycle not containing i involves only non-redirected agents, whose stage-t choices coincide across the two profiles by the induction hypothesis. So such a cycle forms under $(P_i, \tilde{P}_{-i})$ if and only if it forms under (P_i, P_{-i}).

It remains to show that under $(P_i, \tilde{P}_{-i})$ no cycle containing i forms at any stage $t < T$. Suppose one does, and let $t < T$ be the first such stage, with cycle $i \to c_1 \to \cdots \to c_r \to i$: agent i chooses o_{c_1}, agent c_m chooses $o_{c_{m+1}}$ for $m < r$, and agent c_r chooses o_i. At most one of $c_1, \ldots, c_r$ is redirected, and if one is, it is c_r.[8]

Case 1: none of $c_1, \ldots, c_r$ *is redirected.* By the induction hypothesis all these stage-t choices coincide with those under (P_i, P_{-i}). So the same cycle forms at stage t under that profile, contradicting that i is unassigned at stage $T > t$ there.

Case 2: c_r *is redirected.* Then $c_r \in \{w, u\}$, so $o_{c_r} \in \{o', o''\}$ and o_{c_r} is available at the beginning of stage T under (P_i, P_{-i}). Moreover $r \geq 2$. If $r = 1$, then i chooses $o_{c_1} \in \{o', o''\}$ at stage $t < T$, which is excluded by (P2) together with the induction hypothesis. Agents $c_1, \ldots, c_{r-1}$ are not redirected, so by the induction hypothesis their stage-t choices, and that of i, are the same under (P_i, P_{-i}). Applying Lemma OA.5 with $j_0 = i$ and $j_m = c_m$ $(m = 1, \ldots, r)$ under (P_i, P_{-i}), agent i makes no new choice at stage T there, contradicting that I_i' is active.

Hence the same cycles form at stage t under both profiles. So the same agents leave, the same objects are removed, and the set of available objects and the set of active agents at stage $t+1$ coincide. Truthful behavior of a non-redirected agent at stage $t+1$ depends on her preference relation, her previous choice, and the set of available objects, all of which coincide. Each redirected agent is still active with choice o_i, because o_i is removed only through a cycle containing i. ■

By Claim OA.1, i's observations up to stage T are the same under $(P_i, \tilde{P}_{-i})$ as under (P_i, P_{-i}). So truthful play by the others under $\tilde{P}_{-i}$ reaches a history in I_i' and is therefore a behavior of the others consistent with I_i'. Now consider stage T. If i follows her truthful strategy, she chooses o'. Since w chooses o_i, agents i and w form a cycle, and i receives o'. If instead she chooses o'', she receives o'': agents i and u form a cycle if $o'' \neq o_i$, and i forms a self-cycle if $o'' = o_i$. Hence o' is

[8] A redirected agent chooses o_i, so her successor in the cycle is i, and i appears in the cycle only as the successor of c_r.

attainable at I_i', and because choosing o'' instead yields $o'' \neq o'$, whether i receives o' depends on her own behavior, so its receipt is not outside her control. Some object attainable at I_i', namely o', is therefore no longer attainable at I_i''. Since P_i was an arbitrary preference relation consistent with I_i'', the information-revelation condition holds.

Minimality. We show that every action at every active information set is the truthful action of some preference relation consistent with the corresponding history. Given the convention that information sets with one action are suppressed, this yields minimality: no history is off the path of play and no agent ever has a single action.

At stage 1, the action of choosing o_j is the truthful action of every preference relation with peak o_j. Now consider an active information set I_i at a later stage with two actions $a < b$, where the two objects are adjacent, in the current set of available objects, to i's previous and now unavailable choice. Two observations can be made. First, under truthful behavior i's choice is always the best available object according to P_i. This holds at stage 1, and if the best object of the available set is removed, the best object of the new available set is one of its two neighbors. Consequently every object i has chosen is unavailable, and these objects, together with those that became unavailable between them, form an interval containing o_{p_i}. Hence $a < p_i < b$, so a and b lie on opposite sides of the peak. Second, neither a nor b has ever been chosen by i, since every object she chose has been removed.

A single-peaked preference with peak o_p is a linear order that extends the two chains $o_{p-1}\, P\, o_{p-2}\, P \cdots$ and $o_{p+1}\, P\, o_{p+2}\, P \cdots$ with o_p on top. Conversely every such extension is single-peaked. The history at I_i restricts P_i only through the peak and through finitely many comparisons in which an object i chose is preferred to one she rejected. In each of these the preferred object is one i chose, hence lies strictly between a and b. Therefore the history implies neither $a\, P_i\, b$ nor $b\, P_i\, a$: starting from a, only the chain relations apply, and they lead to objects strictly to the left of a, none of which was ever chosen and hence none of which is the preferred element of a comparison; symmetrically for b. Both extensions are consistent with the history, so each of the two actions is the truthful action of some preference relation consistent with I_i. $\square$

Proof that the Gradual TTC has complexity n. For the upper bound, note that at stage 1 an agent has n actions, and at every later active information set she has two. So by Proposition 1 of Nagel and Saitto (2026), complexity is at most n.

For the lower bound, consider agent 1 at her stage-1 information set I_1^1 and the preference relation P_1 given by $o_2 \; P_1 \; o_1 \; P_1 \; o_3 \; P_1 \cdots P_1 \; o_n$. We call an action other than o_1 a *move*. By Lemma OA.3, the only object that can follow the choice of o_1, which forms a self-cycle, is o_1 itself, while every object in O can follow every move. So the best object that can follow any move is the peak o_2, and the worst is o_n. We check that no action is evidently never better than another. The best object that can follow a move is o_2, which is strictly better, according to P_1, than the worst object that can follow any action, namely o_n or o_1. On the other hand, the best object that can follow o_1 is o_1, which is strictly better, according to P_1, than the worst object o_n that can follow every move. Hence the set of all n stage-1 actions contains the truthful action o_2 and is irreducible. □

A.6 Complexity of the Gradual Crawler

For $n \geq 4$ the Gradual Crawler is not an essential frame. As the two lemmas below show, there are consecutive information sets of the same agent between which no payoff-relevant information is revealed. Its complexity must therefore be computed directly. Throughout this subsection, we say that an agent's choice is *standing* at a history if she made it at an earlier stage and, the chosen object being still available and no exchange of hers having intervened, she takes no action while it remains so. By the rules of Section 2.4 of the main text, a standing choice participates in exchanges as a newly made one, so that if the holder of the chosen object chooses hers, the two exchange. One further observation will be used repeatedly. Whether an object is a menu option at an information set depends only on which objects the agent can receive under each of her continuation behaviors, and it is a menu option if and only if it can be received only under behaviors for which it is the unique receivable object.

Our first lemma shows that, for every $n \geq 4$, there is an information set at which the agent has three continuation plans, none of which can be discarded. The configuration is one in which the object the agent has chosen is assigned to someone else while, for some preference relation consistent with what she has observed, the set of attainable objects does not change, so that nothing is revealed and her plan must specify her next decision as well.

Lemma OA.6. *Let $n \geq 4$ and consider the following first-stage choices. Agents $1, \ldots, n-4$ (none when $n = 4$) and agent $n-2$ choose their endowments. Agent $n-3$ chooses o_{n-2}, agent $n-1$ chooses o_{n-2}, and agent n chooses o_{n-1}. Let I_0 denote the information set at which agent $n-3$ is called upon at the second stage, holding o_{n-3} and observing $O^h = \{o_{n-3}, o_{n-1}, o_n\}$. Then the following statements hold.*

(i) *Every history in I_0 induces the same profile of holdings, directions, and standing choices. Specifically, agent $n-1$ holds o_{n-1} with $d_{n-1} = -$ and is called upon. Agent n holds o_n with $d_n = -$ and standing choice o_{n-1}. Every other agent has left with her endowment.*

(ii) *No payoff-relevant information is revealed between I_0 and the information set I_1 reached when agent $n-1$ keeps, at which agent $n-3$ still holds o_{n-3} and observes $O^h = \{o_{n-3}, o_n\}$. By contrast, payoff-relevant information is revealed between I_0 and the information set reached after agent $n-3$ exchanges.*

(iii) *Agent $n-3$ has three continuation plans at I_0: keep (D_0); continue at I_0 and keep at I_1 (D_1); continue at both (D_2).*

(iv) *There are no menu options at I_0. Moreover, for the single-peaked preference relation $P^\dagger$ with peak o_{n-1} and $o_{n-3} \, P^\dagger \, o_n$, the set $\{D_0, D_1, D_2\}$ contains her truthful plan and is irreducible.*

Consequently, the strategic complexity of the Gradual Crawler is at least 3 for every $n \geq 4$.

Proof. *(i)* Fix a history in I_0. Since agent $n-3$ holds her endowment at the second stage, she did not exchange. The objects $o_1, \ldots, o_{n-4}$ and o_{n-2} were assigned at the first stage, while o_{n-1} and o_n were not. Agent n's only actions are o_{n-1} and her endowment. Keeping would remove o_n, so she chose o_{n-1}.

Agent $n-1$ could not keep, as this would remove o_{n-1}. She also could not choose o_n. Otherwise, she and agent n would have chosen each other's objects and exchanged. Agent $n-1$ would then hold o_n moving upward, her assignment would be final, and o_n would be removed. Hence agent $n-1$ chose o_{n-2}.

Agent $n-2$ could not choose o_{n-3}. Otherwise, she and agent $n-3$ would have chosen each other's objects and exchanged, contradicting $e^h(n-3) = o_{n-3}$. She also could not choose o_{n-1}. Since agent $n-1$ chose o_{n-2}, the two would then have chosen each other's objects and exchanged. Agent $n-1$ would hold o_{n-2} moving downward. Since o_{n-3} remains available, her assignment would not be final, so o_{n-2} would not be removed. Hence agent $n-2$ kept.

Finally, consider $j \leq n-4$. No exchange between agents j and $j+1$ is possible. For $j+1 = n-3$ this is because agent $n-3$ did not exchange. For $j+1 \leq n-4$, agent j would hold o_{j+1} after the exchange, moving upward. Since o_{n-3} remains available above o_{j+1}, her assignment would not be final. Then o_{j+1} would remain unassigned, a contradiction. Nor could agent j choose o_{j+1} without the corresponding exchange. She would then remain in the procedure holding o_j, so o_j would not be assigned. Each agent $j \leq n-4$ therefore either kept her endowment or, if $j \geq 2$, chose o_{j-1}. In the latter case, all objects below o_j were removed within the stage, and she received o_j as a final assignment. In every case agent j leaves with o_j, so all histories in I_0 induce the stated profile. Agent n's standing choice o_{n-1} persists because o_{n-1} remains available.

(ii) Suppose agent $n-3$ continues at I_0, choosing o_{n-1}, while agent $n-1$ keeps. Then o_{n-1} is removed and agent n's standing choice disappears. Agent $n-3$ is called upon at I_1 with $O^h = \{o_{n-3}, o_n\}$. By the same reasoning as in (i), every history in I_1 arises from the profile of (i) followed by agent $n-1$ keeping. At I_1, agent n is therefore called upon as well, with actions o_n and o_{n-3}.

Consider the preference relation P^* of agent $n-3$ with peak o_n, that is, $o_n \, P^* \, o_{n-1} \, P^* \cdots P^* \, o_1$. It is consistent with I_1. We claim that the objects attainable for P^* at I_0 and at I_1 are in both cases $\{o_{n-3}, o_n\}$. Then no attainable object is lost, and the revelation condition fails at P^*. Under her truthful strategy she continues at every decision.

Object o_{n-3} is attainable at I_0. Let agent $n-1$ keep, and then let agent n keep at the next stage. Keeping is truthful for agent $n-1$ if her peak is o_{n-2} and she ranks o_{n-1} above o_{n-3}. Keeping is truthful for agent n if her peak is o_{n-1} and ranks o_n above o_{n-3}. After these choices, o_n is removed and agent $n-3$'s assignment is final at o_{n-3}. Object o_n is attainable at I_0 as well. Let agent $n-1$ continue, which is truthful for a preference relation with peak at or below o_{n-3}.

Agents $n-3$ and $n-1$ then exchange. Agent n's standing choice o_{n-1} persists, since o_{n-1} is now held by agent $n-3$. It makes the next exchange certain when agent $n-3$ continues. She then receives o_n, which is the largest available object, so her assignment is final.

Both objects also remain receivable at I_1 under the truthful strategy of P^*, whatever agent n does. If agent n keeps, agent $n-3$ is final at o_{n-3}. If agent n continues, the two exchange and she receives o_n. Since each of these choices of agent n is truthful for some consistent preference relation, as above, both objects are attainable at I_1 as well. Finally, no other object is attainable for P^* at I_0 or I_1. Objects below o_{n-3}, and objects assigned at the first stage, can never be received. Object o_{n-1} can be received only after an exchange with agent $n-1$. From then on, the truthful strategy of P^* continues, and the certain trade with agent n delivers o_n, so P^* never ends at o_{n-1}. Hence the attainable objects are exactly $\{o_{n-3}, o_n\}$ at both information sets, and no payoff-relevant information is revealed.

By contrast, suppose agent $n-3$ exchanges at I_0. Denote by J the information set at which she is next called upon, holding o_{n-1}. Agent $n-1$ then holds o_{n-3} moving downward. Every object below o_{n-3} is assigned, so her assignment is final and $O^J = \{o_{n-1}, o_n\}$. Object o_{n-3} is attainable at I_0 for every preference relation consistent with J. Such a relation ranks o_{n-1} above o_{n-3}, so its peak is o_{n-2}, o_{n-1}, or o_n. Consider again the play in which agent $n-1$ keeps and then agent n keeps. If her truthful strategy keeps at I_1, she receives o_{n-3}. If it continues at I_1, then o_n is removed and she is final at o_{n-3}. Either way she receives o_{n-3}. At J and beyond, she holds o_{n-1}. Her direction is fixed, so she can never again receive o_{n-3}. So payoff-relevant information is revealed between I_0 and J, and, as noted in Section A.1, between I_0 and everything beyond J.

(iii) By (ii), two continuation behaviors belong to the same plan at I_0 if and only if they agree at I_0 and at I_1. Keeping is absorbing, so the plans are the three listed.

(iv) The objects agent $n-3$ can receive from I_0 are $\{o_{n-3}, o_{n-1}, o_n\}$. By the second convention, the only object that can follow D_0 is o_{n-3}. The plans D_1 and D_2 take the same action at I_0, and all three objects can follow either. Indeed, after continuing, she receives o_{n-1} if agent $n-1$ continues and she then keeps. She

receives o_n if agent $n-1$ continues and she continues again, since the standing choice of agent n makes the trade certain. Finally, she receives o_{n-3} if agent $n-1$ keeps and she then keeps at I_1.

There are no menu options at I_0. Consider the behavior that continues at I_0 and keeps at I_1 and at J. It can end in o_{n-3}, if agent $n-1$ keeps, and in o_{n-1}, if she continues. So each of these two objects can be received under a behavior that does not receive it with certainty. Consider next the behavior that continues at I_0, keeps at I_1, and continues at J. It can end in o_{n-3} or in o_n, so the same holds for o_n. Hence no object is disregarded in any comparison.

Now take the single-peaked preference relation $P^\dagger$ with peak o_{n-1} and $o_{n-3} \, P^\dagger \, o_n$. Single-peakedness pins down $o_{n-1} \, P^\dagger \, o_{n-2} \, P^\dagger \, o_{n-3}$ and leaves the comparison of o_{n-3} with o_n free, so $P^\dagger$ is admissible. It is consistent with I_0, since its truthful stage-1 choice from endowment o_{n-3} is o_{n-2}. Its truthful plan is D_1: it continues at I_0, since o_{n-1} is the peak, and keeps at I_1, since $o_{n-3} \, P^\dagger \, o_n$. We check every ordered pair of plans. D_0 is not evidently never better than D_1 or D_2: the best object that can follow D_0 is o_{n-3}, the worst that can follow D_1 or D_2 is o_n, and $o_{n-3} \, P^\dagger \, o_n$. Neither D_1 nor D_2 is evidently never better than D_0: their best object is o_{n-1}, the worst that can follow D_0 is o_{n-3}, and $o_{n-1} \, P^\dagger \, o_{n-3}$. And neither of D_1, D_2 is evidently never better than the other, since the same objects can follow either, and the best of them, o_{n-1}, is strictly preferred to the worst, o_n. The set $\{D_0, D_1, D_2\}$ contains the truthful plan and is irreducible. □

Our second lemma shows that at $n = 5$ there is an information set at which the agent has five continuation plans, none of which can be discarded. Here a standing choice of another agent names the object the agent currently holds. So her next exchange is certain whenever she continues, and her truthful play from the information set onward is deterministic. On its own, this keeps the agent's problem simple: each continuation plan then guarantees the object at which it stops, every receivable object is a menu option, and every plan but the truthful one can be discarded. What creates the five plans below is that an exchange elsewhere on the line splits the continuation into two information sets the agent can tell apart. A plan that stops at one and continues at the other no longer guarantees where it ends, so the objects in question are not menu options, and no plan can be discarded.

Lemma OA.7. *Let $n = 5$ and consider the following first-stage choices. Agent 1 chooses o_2. Agent 2 chooses o_3 and agent 3 chooses o_2, so the two exchange. Agent 4 chooses o_3, and agent 5 chooses o_4. Let I^* denote the information set at which agent 2 is called upon at the second stage, holding o_3 and observing $O^h = \{o_1, \ldots, o_5\}$. Then the following statements hold.*

- *(i) I^* is a singleton. Specifically, agent 1 holds o_1 with standing choice o_2. Agent 3 holds o_2 with $d_3 = -$ and is called upon. Agent 4 holds o_4 with standing choice o_3. Agent 5 holds o_5 with standing choice o_4.*
- *(ii) If agent 2 continues at I^*, the exchange with agent 4 occurs with certainty, and she is next called upon holding o_4 at one of two information sets. If agent 3 keeps, she is at J, with $O^h = \{o_1, o_3, o_4, o_5\}$. If agents 3 and 1 exchange, she is at J', with $O^h = \{o_2, o_3, o_4, o_5\}$. At either, if she continues, the exchange with agent 5 occurs with certainty and her assignment is final at o_5.*
- *(iii) No payoff-relevant information is revealed between I^* and J, nor between I^* and J'. Consequently, agent 2 has exactly five continuation plans at I^*: keep, and the four combinations of keep or continue at J and at J'.*
- *(iv) The unique menu option at I^* is o_3, and it can be received only under the keep plan. Moreover, for the single-peaked preference relation $P^\dagger$ given by $o_4 \, P^\dagger \, o_3 \, P^\dagger \, o_2 \, P^\dagger \, o_1 \, P^\dagger \, o_5$, the set of all five plans contains her truthful plan and is irreducible.*

Consequently, the strategic complexity of the Gradual Crawler at $n = 5$ is at least 5.

Proof. (i) Since all five objects remain available at the second stage, no agent kept and no assignment became final at the first stage. Agent 2 holds o_3, so she exchanged. Her partner is the first-stage holder of o_3, namely agent 3. Hence agent 3 chose o_2. Agent 1's only action other than keeping is o_2, and agent 5's only action other than keeping is o_4. So both choices are pinned down. Agent 4 chose o_3 or o_5. Suppose she chose o_5. She and agent 5 would then have chosen each other's objects and exchanged. Agent 4 would hold o_5 moving upward, her assignment would be final, and o_5 would be removed, a contradiction. Hence agent

4 chose o_3. Her choice is standing at the second stage because o_3 remains available, held by agent 2.

(ii) Agent 4's standing choice names o_3, agent 2's holding. The rules give agent 4 no move while o_3 remains available. If agent 2 continues, she names o_4, agent 4's holding. The two have then chosen each other's objects, so the exchange occurs whatever the others do. In the same stage, agent 3 either keeps or continues. If she keeps, o_2 is removed and agent 1's standing choice disappears. If she continues, she names o_1, agent 1's holding. Agent 1's standing choice names o_2, agent 3's holding, so the two exchange. Agent 3 then holds o_1 moving downward, and her assignment is final. Object o_1 is removed, while agent 1 remains in the procedure holding o_2. Agent 2, having exchanged, is called upon at the next stage. She observes one of the two sets displayed in the statement. As in (i), each of J and J' is induced by a unique continuation of I^*. At either, agent 5's standing choice names o_4, agent 2's holding. So if agent 2 continues, the two exchange. She then holds o_5, the largest available object, so her assignment is final.

(iii) Consider the preference relations of agent 2 consistent with J, or equivalently with J'. Each chose o_3 at the first stage, so its peak is o_3 or above. Each continued at I^*, so it ranks o_4 above o_3. Its peak is therefore o_4 or o_5. By (ii), agent 2's assignment under joint truthful play from I^* is deterministic. A relation with peak o_4 continues into o_4 with certainty and then keeps, receiving o_4. The relation with peak o_5 continues twice and receives o_5. Agent 3's truthful choice routes play to J or to J' but does not affect agent 2's assignment. Hence, for every consistent preference relation, exactly one object is attainable at I^*: o_4 or o_5. The same object remains receivable at J and at J' under her truthful strategy, whatever the others do: keeping yields o_4, and continuing yields o_5 with certainty. So no attainable object is lost for any consistent preference relation, and no payoff-relevant information is revealed between I^* and J, nor between I^* and J'. Two continuation behaviors therefore belong to the same plan at I^* if and only if they agree at I^*, at J, and at J'. Keeping is absorbing, and no decision of agent 2 follows J or J'. So the plans are the five listed.

(iv) The objects agent 2 can receive from I^* are $\{o_3, o_4, o_5\}$. By the second convention, the only object that can follow the keep plan is o_3. The four continuation plans take the same action at I^*, and the objects that can follow each of them

are $\{o_4, o_5\}$. Indeed, once she continues, the certain exchange moves her holding strictly upward, so o_3 can no longer be received.

Her continuation behaviors admit the following possibilities. Keeping can end only in o_3. Continuing and then keeping at both J and J' can end only in o_4. Continuing and then continuing at both can end only in o_5. Continuing, then keeping at one of J and J' and continuing at the other, can end in o_4 or in o_5. Hence o_3 is a menu option: it can be received only under behaviors that receive it with certainty. Objects o_4 and o_5 are not menu options: each can be received under a behavior of the last kind, which does not receive it with certainty. The menu option o_3 can be received only under the keep plan. The keep plan can end only in o_3 itself. So no object is disregarded in any comparison.

Now consider the preference relation $P^\dagger$. It is single-peaked: its peak is o_4, and the comparison of o_5 with the objects on the other side of the peak is free. Its truthful stage-1 choice from endowment o_2 is o_3, it continues at I^* since o_4 is the peak, and it keeps at J and at J'. Hence $P^\dagger$ is consistent with J and with J', and its truthful plan is the one that continues at I^* and keeps at both. We check every ordered pair of plans. The keep plan is not evidently never better than any continuation plan: its best object is o_3, the worst that can follow a continuation plan is o_5, and $o_3 \, P^\dagger \, o_5$. No continuation plan is evidently never better than the keep plan: its best object is o_4, the worst that can follow keep is o_3, and $o_4 \, P^\dagger \, o_3$. And no continuation plan is evidently never better than another, since the same objects can follow each of them and $o_4 \, P^\dagger \, o_5$. The set of all five plans contains the truthful plan and is irreducible. This proves the final claim of the lemma. □

Proposition OA.2. *The strategic complexity of the Gradual Crawler is* 2 *for* $n = 3$*,* 3 *for* $n = 4$*, and* 5 *for* $n = 5$*. For every* $n \geq 4$ *it is at least* 3.

Proof. We first show that payoff-relevant information is revealed between the first-stage information set of any agent i and each of her subsequent information sets. As noted in Section A.1, it suffices to consider immediate successors. There are two kinds. Suppose first that the successor follows an exchange, so that there she no longer holds o_i. Her endowment is attainable at the first stage for every preference relation. To see this, let every other agent choose her own endowment at stage 1, which is truthful when her peak is her endowment. All other objects are then removed at the first stage, and agent i's assignment is final at o_i whatever her truthful action. After an exchange, her direction is fixed and her holding moves strictly away from o_i. So the endowment can never be received again. Suppose instead that the successor is reached without an exchange. Then her stage-1 choice, say o_{i+1}, has been assigned to another agent while she still holds o_i; the case of o_{i-1} is symmetric. Every preference relation consistent with this successor chose o_{i+1} at stage 1. Moreover, o_{i+1} is attainable at the first stage for each of them. To see this, let agent $i+1$ choose o_i, which is truthful when her peak lies weakly below o_i, and let every agent $j > i+1$ choose her own endowment. Agents i and $i+1$ have then chosen each other's objects, so they exchange. All objects above o_{i+1} are removed at the first stage, and agent i's assignment is final at o_{i+1}. At the successor, o_{i+1} has been assigned to another agent and can never be received. Hence, in both cases, some attainable object is lost for every consistent preference relation. In particular, at every first-stage information set, plans coincide with actions.

We next establish the lower bounds. For $n \geq 4$ the bound of 3 is Lemma OA.6, and for $n = 5$ the bound of 5 is Lemma OA.7. For $n = 3$, consider agent 1 at her first-stage information set I with the preference relation P_1 given by $o_2 \, P_1 \, o_1 \, P_1 \, o_3$. By the previous paragraph, her plans at I are her two actions, keep and choose o_2. We have $\mathcal{O}_1(I, o_1) = \{o_1\}$ and $\mathcal{O}_1(I, o_2) = \{o_1, o_2, o_3\}$. Indeed, she receives o_2 if agent 2 chooses o_1 and she then keeps. She receives o_1 if agent 2 keeps and she then keeps the object she holds. She receives o_3 if agent 2 chooses o_1, she continues, and agent 3's stage-1 choice of o_2 stands. The exchange then gives her o_3, which is final. No object is a menu option at I. The behavior that chooses o_2 and thereafter keeps whenever called can end in o_1 or in o_2. The behavior that chooses o_2 and thereafter continues whenever called can end in o_1 or in o_3. So each receivable object can be received under a behavior that does not receive it with certainty. Neither action is evidently never better than the other. The best object that can follow the choice of o_2 is o_2, and $o_2 \, P_1 \, o_1$, the only object that can follow keep. The best object that can follow keep is o_1, and $o_1 \, P_1 \, o_3$, the worst that can follow the choice of o_2. The two-element set contains the truthful action and is irreducible, so complexity at $n = 3$ is at least 2.

It remains to establish the upper bounds. Fix $n \in \{3, 4, 5\}$. The Gradual

Crawler with n agents is a finite game form: each agent has finitely many information sets, finitely many single-peaked preference relations are consistent with each, and finitely many continuation behaviors are available at each. Consequently, every notion of Section A.1 involves only finitely many cases: which preference relations are consistent with an information set, which objects are attainable there, whether payoff-relevant information is revealed between two information sets, what the continuation plans and the menu options are, and which sets of plans are irreducible. Each can therefore be verified directly from the definitions, with the objects outside an agent's control given by the second convention. Every information set is reached under the truthful strategy profile. The largest irreducible sets contain 2, 3, and 5 plans for $n = 3$, 4, and 5, and for $n = 4$ and $n = 5$ they arise at the information sets of Lemma OA.6 and Lemma OA.7. $\square$

Remark OA.1. For $n \geq 6$ we do not know the exact value. The sequences of certain exchanges lengthen with n and can split in more ways, but larger available sets also pool more histories into each information set, which can restore information revelation, and, wherever the continuation does not split, every receivable object is a menu option and every plan but the truthful one can be discarded. The verification underlying Proposition OA.2 can be carried out for moderate values of n directly.

Remark OA.2. The Gradual Crawler is an essential frame if and only if $n = 3$. The failures for $n \geq 4$ are proved in Lemma OA.6(ii) and, at $n = 5$, in Lemma OA.7(iii). For $n = 3$, the case-by-case check behind Proposition OA.2 also confirms that payoff-relevant information is revealed between every pair of consecutive information sets of the same agent, and that every action at every information set is the truthful action of some consistent preference relation, so that the game tree is minimal.

Remark OA.3. The value of five in Proposition OA.2 does not contradict Theorem 6 of Nagel and Saitto (2026), which establishes an upper bound of two for the complexity of obviously strategy-proof essential frames in rich environments, for two reasons. First, our environment is not rich, and their Theorem C.1, which extends the bound to single-peaked preferences in the sense of Arribillaga et al. (2020), concerns the choice of a single social alternative from a linearly ordered set, a setting distinct from our reallocation model. Second, and more importantly, both results are confined to essential frames, and the Gradual Crawler is not one for $n \geq 4$ (Remark OA.2).

Remark OA.4. Our comparison concerns the mechanisms of Section 2 of the main text on the single-peaked domain and is therefore distinct from Theorem 4 of Nagel and Saitto (2026), which compares their TTC with partial feedback to the static TTC on the unrestricted domain and obtains a strict ranking already for three agents. Two features account for the different threshold. First, the single-peaked domain shrinks the static report space from $n!$ to 2^{n-1}, lowering the complexity of both static mechanisms. Second, the adjacency structure of our gradual implementation makes the same objects receivable after every first choice on the same side, increasing the complexity of the Gradual TTC relative to a mechanism in which each first choice clinches more.

B Preference Profiles

Each subject is endowed with the good whose identifier matches her own. The allocations under truthful reporting are shown in red.

Table B.1: Profile 1

	P_1	P_2	P_3	P_4	P_5
Rank 1	2	5	1	5	4
Rank 2	3	4	2	4	3
Rank 3	1	3	3	3	5
Rank 4	4	2	4	2	2
Rank 5	5	1	5	1	1

Table B.2: Profile 2

	P_1	P_2	P_3	P_4	P_5
Rank 1	4	4	2	3	4
Rank 2	3	3	1	2	3
Rank 3	5	5	3	4	2
Rank 4	2	2	4	1	1
Rank 5	1	1	5	5	5

Table B.3: Profile 3

	P_1	P_2	P_3	P_4	P_5
Rank 1	2	3	2	5	3
Rank 2	3	4	3	4	2
Rank 3	4	2	4	3	1
Rank 4	5	1	5	2	4
Rank 5	1	5	1	1	5

Table B.4: Profile 4

	P_1	P_2	P_3	P_4	P_5
Rank 1	5	3	4	2	1
Rank 2	4	2	5	1	2
Rank 3	3	4	3	3	3
Rank 4	2	5	2	4	4
Rank 5	1	1	1	5	5

Table B.5: Profile 5

	P_1	P_2	P_3	P_4	P_5
Rank 1	3	1	5	3	4
Rank 2	4	2	4	4	3
Rank 3	5	3	3	2	5
Rank 4	2	4	2	1	2
Rank 5	1	5	1	5	1

Table B.6: Profile 6

	P_1	P_2	P_3	P_4	P_5
Rank 1	2	1	5	1	4
Rank 2	1	2	4	2	3
Rank 3	3	3	3	3	5
Rank 4	4	4	2	4	2
Rank 5	5	5	1	5	1

Table B.7: Profile 7

	P_1	P_2	P_3	P_4	P_5
Rank 1	4	3	4	3	1
Rank 2	3	4	5	2	2
Rank 3	2	5	3	1	3
Rank 4	1	2	2	4	4
Rank 5	5	1	1	5	5

Table B.8: Profile 8

	P_1	P_2	P_3	P_4	P_5
Rank 1	3	4	1	5	1
Rank 2	4	3	2	4	2
Rank 3	2	5	3	3	3
Rank 4	1	2	4	2	4
Rank 5	5	1	5	1	5

Table B.9: Profile 9

	P_1	P_2	P_3	P_4	P_5
Rank 1	3	5	4	1	1
Rank 2	2	4	3	2	2
Rank 3	1	3	2	3	3
Rank 4	4	2	1	4	4
Rank 5	5	1	5	5	5

Table B.10: Profile 10

	P_1	P_2	P_3	P_4	P_5
Rank 1	3	5	5	1	3
Rank 2	2	4	4	2	2
Rank 3	1	3	3	3	4
Rank 4	4	2	2	4	1
Rank 5	5	1	1	5	5

Table B.11: Profile 11

	P_1	P_2	P_3	P_4	P_5
Rank 1	5	5	2	2	3
Rank 2	4	4	1	1	2
Rank 3	3	3	3	3	1
Rank 4	2	2	4	4	4
Rank 5	1	1	5	5	5

Table B.12: Profile 12

	P_1	P_2	P_3	P_4	P_5
Rank 1	5	1	4	3	2
Rank 2	4	2	5	2	1
Rank 3	3	3	3	1	3
Rank 4	2	4	2	4	4
Rank 5	1	5	1	5	5

Table B.13: Profile 13

	P_1	P_2	P_3	P_4	P_5
Rank 1	4	5	1	3	3
Rank 2	3	4	2	4	2
Rank 3	2	3	3	2	4
Rank 4	1	2	4	5	5
Rank 5	5	1	5	1	1

Table B.14: Profile 14

	P_1	P_2	P_3	P_4	P_5
Rank 1	3	1	2	5	1
Rank 2	2	2	1	4	2
Rank 3	4	3	3	3	3
Rank 4	5	4	4	2	4
Rank 5	1	5	5	1	5

Table B.15: Profile 15

	P_1	P_2	P_3	P_4	P_5
Rank 1	2	1	5	2	4
Rank 2	3	2	4	1	3
Rank 3	1	3	3	3	2
Rank 4	4	4	2	4	5
Rank 5	5	5	1	5	1

Table B.16: Profile 16

	P_1	P_2	P_3	P_4	P_5
Rank 1	5	3	2	3	3
Rank 2	4	2	1	2	2
Rank 3	3	4	3	1	4
Rank 4	2	5	4	4	5
Rank 5	1	1	5	5	1

Table B.17: Profile 17

	P_1	P_2	P_3	P_4	P_5
Rank 1	3	5	1	1	3
Rank 2	4	4	2	2	4
Rank 3	2	3	3	3	2
Rank 4	5	2	4	4	5
Rank 5	1	1	5	5	1

Table B.18: Profile 18

	P_1	P_2	P_3	P_4	P_5
Rank 1	2	3	4	2	2
Rank 2	3	4	5	1	3
Rank 3	4	5	3	3	1
Rank 4	5	2	2	4	4
Rank 5	1	1	1	5	5

Table B.19: Profile 19

	P_1	P_2	P_3	P_4	P_5
Rank 1	3	5	5	2	1
Rank 2	4	4	4	1	2
Rank 3	5	3	3	3	3
Rank 4	2	2	2	4	4
Rank 5	1	1	1	5	5

Table B.20: Profile 20

	P_1	P_2	P_3	P_4	P_5
Rank 1	3	3	4	3	1
Rank 2	4	4	5	4	2
Rank 3	2	5	3	2	3
Rank 4	5	2	2	5	4
Rank 5	1	1	1	1	5

C English translation of the instructions, quiz, and decision screens for four treatments.

C.1 Gradual Crawler

C.1.1 Instructions

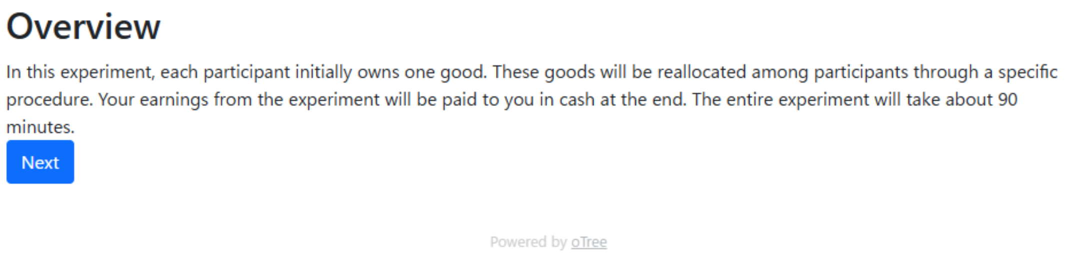

Figure C.1: Gradual Crawler: Instruction 1

Initial Setup

You will be randomly assigned to a group of five participants. At the beginning, you will see a screen showing the good you initially own and the points you receive from each good. The higher the points, the more desirable the good is for you.
The screen below is provided as an example.
In this example, the participant initially owns Good 3.
She receives 3 points if assigned Good 1, 7 points if assigned Good 2, 9 points if assigned Good 3, 5 points if assigned Good 4, and 1 point if assigned Good 5.

- There are 5 goods.
- The points you receive from each good are shown in the table on the right. The good you initially own is highlighted in red.
- Please select a good, using the box below. You will only be shown goods you can choose from.

Good	Points
1	3
2	7
3	9
4	5
5	1

The good you are asking for -- Submit

This is an example screenshot.

Next

Figure C.2: Gradual Crawler: Instruction 2

Reallocation Procedure

Goods are indexed from 1 to 5.
Each participant chooses one good. She may choose either her good or one of its adjacent goods. Adjacent goods are those directly before or after her good in the index. Note that Goods 1 and 5 each have only one adjacent good: Good 2 is adjacent to Good 1, and Good 4 is adjacent to Good 5.
Your final assignment will be determined by a procedure that takes into account (i) the good you choose, (ii) the initial assignments, and (iii) the choices of the other participants. The procedure takes place over multiple stages, as described below.

Next

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Figure C.3: Gradual Crawler: Instruction 3

Reallocation Procedure

If a participant chose her current good, her assignment is finalized and she is assigned that good.
If two participants chose each other's good, they swap. After a swap, the assignment is finalized only if the new good has the smallest or largest index among the remaining goods. Otherwise, the assignment is not yet final, and the participant remains in the procedure while holding her new good.
The procedure then moves to the next stage.

Next

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Figure C.4: Gradual Crawler: Instruction 4

Reallocation Procedure

If a participant did not swap in the previous stage and the good she chose is still available, no action is needed.
Otherwise, she must choose again, continuing in the same direction as before.
• If her previous choice was toward a smaller-indexed good, she may choose either her current good or its adjacent available good with a smaller index.
• If her previous choice was toward a larger-indexed good, she may choose either her current good or its adjacent available good with a larger index.

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Figure C.5: Gradual Crawler: Instruction 5

Reallocation Procedure

If a participant chose her current good, her assignment is finalized and she is assigned that good.
If two participants chose each other's good, they swap. After a swap, the assignment is finalized only if the new good has the smallest or largest index among the remaining goods. Otherwise, the assignment is not yet final, and the participant remains in the procedure while holding her new good.
The procedure then moves to the next stage and repeats the same process until every participant is assigned a good.

Next

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Figure C.6: Gradual Crawler: Instruction 6

Example 1

Consider an example with three participants. In the actual experiment, there are five participants.
The initial assignments are: Participant 1 with Good 1, Participant 2 with Good 2, and Participant 3 with Good 3. Suppose all three participants choose Good 2.
Since Participant 2 chose her own good, Good 2 is assigned to her. Because Good 2 is already assigned, Participants 1 and 3 must choose again. The remaining goods are Good 1 and Good 3, both adjacent to Good 2. Therefore, Participant 1 can choose either Good 1 or Good 3, and Participant 3 can choose either Good 3 or Good 1.
Suppose Participant 1 chooses Good 3 and Participant 3 chooses Good 1. Since they chose each other's good, they swap. Participant 1 is assigned Good 3 and Participant 3 is assigned Good 1. Both goods have either the smallest or largest index among the remaining goods, so these assignments are finalized.
As all participants are now assigned a good, the procedure ends.

Next

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Figure C.7: Gradual Crawler: Instruction 7

Example 2

Consider another example with three participants.
The initial assignments are: Participant 1 with Good 1, Participant 2 with Good 2, and Participant 3 with Good 3. Suppose Participant 1 chooses Good 2, Participant 2 chooses Good 1, and Participant 3 chooses Good 2.
Since Participants 1 and 2 chose each other's good, they swap. Because Good 1 has the smallest index, Participant 2's assignment is finalized.
Since Participant 1 now owns a new good, she must choose a good again. She may choose either Good 2 or Good 3, the higher-indexed adjacent good of Good 2. Suppose she chooses Good 3.
Since Participant 3 did not swap in the previous stage and the good she chose is still available, no action is needed.
Since Participants 1 and 3 chose each other's good, they swap. Both Good 2 and Good 3 have either the smallest or largest index among the remaining goods, so their assignments are finalized.
As all participants are now assigned a good, the procedure ends.

Next

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Figure C.8: Gradual Crawler: Instruction 8

Rounds and Payment

You will participate in this type of allocation problem for 20 rounds. In each round, you will have a different initial good and different points for each good, and you will play with different participants.
You will receive a show-up fee of ¥750. At the end of the experiment, one round will be randomly selected, and your total earnings will be based on your earnings from that round. The conversion rate is 1 point = ¥250.

Next

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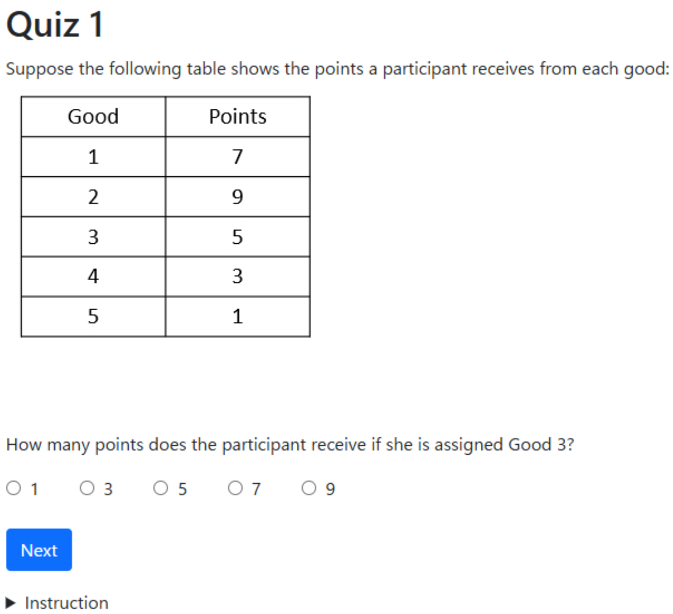
Figure C.9: Gradual Crawler: Instruction 9

C.1.2 Quiz



Quiz 1

Suppose the following table shows the points a participant receives from each good:

Good	Points
1	7
2	9
3	5
4	3
5	1

How many points does the participant receive if she is assigned Good 3?

○ 1 ○ 3 ○ 5 ○ 7 ○ 9

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▶ Instruction

Figure C.10: Gradual Crawler: Question 1

Quiz 2

There are 5 goods, and a participant initially owns Good 2.

(1) The participant asked for Good 3 in stage 1 but did not receive it. The goods that remain available in stage 2 are Goods 1, 2, 4, and 5. Will the participant need to ask for a good in stage 2? If so, which good(s) can the participant ask for in stage 2?

☐ No need to ask ☐ Good 1 ☐ Good 2 ☐ Good 3 ☐ Good 4 ☐ Good 5

(2) The participant asked for Good 3 in stage 1 but did not receive it. All 5 goods remain available in stage 2. Will the participant need to ask for a good in stage 2? If so, which good(s) can the participant ask for in stage 2?

☐ No need to ask ☐ Good 1 ☐ Good 2 ☐ Good 3 ☐ Good 4 ☐ Good 5

(3) The participant asked for Good 3 in stage 1 and received it. All 5 goods remain available in stage 2. Will the participant need to ask for a good in stage 2? If so, which good(s) can the participant ask for in stage 2?

☐ No need to ask ☐ Good 1 ☐ Good 2 ☐ Good 3 ☐ Good 4 ☐ Good 5

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Figure C.11: Gradual Crawler: Question 2

Quiz 3

There are 5 goods and 5 participants, and each of the 5 participants initially owns one of the 5 goods: Participant 1 owns Good 1, Participant 2 owns Good 2, Participant 3 owns Good 3, Participant 4 owns Good 4, and Participant 5 owns Good 5.

In stage 1, 5 participants asked for the following goods, respectively:

Participant 1: Good 2   Participant 2: Good 2   Participant 3: Good 4
Participant 4: Good 3   Participant 5: Good 4

(1) Is Participant 2's assignment **finalized** in stage 1? If so, which good is permanently assigned?

○ Not finalized ○ Good 1 ○ Good 2 ○ Good 3 ○ Good 4 ○ Good 5

(2) Is Participant 1's assignment **finalized** in stage 1? If so, which good is permanently assigned?

○ Not finalized ○ Good 1 ○ Good 2 ○ Good 3 ○ Good 4 ○ Good 5

(3) Which pair of participants **swap** their goods in stage 1?

○ Participants 3 and 4 ○ Participants 4 and 5 ○ Participants 3 and 5 ○ None of them

Next

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Figure C.12: Gradual Crawler: Question 3

C.1.3 Main screens

Round1

· There are 5 goods.
· The points you receive from each good are shown in the table on the right. The good you initially own is highlighted in red.
· Please select a good, using the box below. You will only be shown goods you can choose from.

The good you are asking for 4 Submit

Good	Points
1	9
2	7
3	5
4	3
5	1

Figure C.13: Gradual Crawler: Main decision screen 1

Round1

· The good you currently own is highlighted in red.
· Goods that are no longer available are grayed out.
· Please select a good, using the box below. You will only be shown goods you can choose from.

Good	Points
1	9
2	7
3	5
4	3
5	1

The good you are asking for 3 Submit

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Figure C.14: Gradual Crawler: Main decision screen 2. Shown when necessary

Round1

· The good you currently own is highlighted in red.
· Goods that are no longer available are grayed out.
· Please select a good, using the box below. You will only be shown goods you can choose from.

Good	Points
1	9
2	7
3	5
4	3
5	1

The good you are asking for 5 Submit

Figure C.15: Gradual Crawler: Main decision screen 3. Shown when necessary

Round1 result

· You are assigned Good 5.
· The table showing the points you receive from each good is displayed in the left panel, while the list of goods you have requested is displayed in the right panel as a summary.

Good	Points
1	9
2	7
3	5
4	3
5	1

Stage 1	4
Stage 2	3
Stage 3	5

Next

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Figure C.16: Gradual Crawler: End of a round feedback

C.2 Gradual TTC

C.2.1 Instructions

Overview

In this experiment, each participant initially owns one good. These goods will be reallocated among participants through a specific procedure. Your earnings from the experiment will be paid to you in cash at the end. The entire experiment will take about 90 minutes.

Next

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Figure C.17: Gradual TTC: Instruction 1

Initial Setup

You will be randomly assigned to a group of five participants. At the beginning, you will see a screen showing the good you initially own and the points you receive from each good. The higher the points, the more desirable the good is for you.
The screen below is provided as an example.
In this example, the participant initially owns Good 3.
She receives 3 points if assigned Good 1, 7 points if assigned Good 2, 9 points if assigned Good 3, 5 points if assigned Good 4, and 1 point if assigned Good 5.

- There are 5 goods.
- The points you receive from each good are shown in the table on the right. The good you initially own is highlighted in red.
- Please select a good, using the box below. You will only be shown goods you can choose from.

Good	Points
1	3
2	7
3	9
4	5
5	1

The good you are asking for -- Submit

This is an example screenshot.

Next

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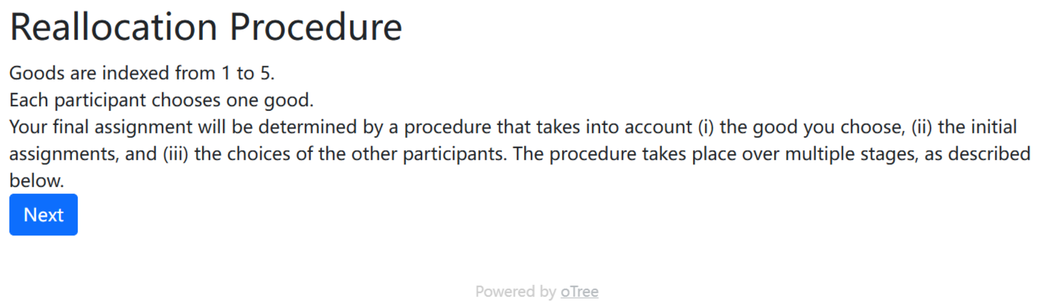
Figure C.18: Gradual TTC: Instruction 2



Reallocation Procedure

Goods are indexed from 1 to 5.
Each participant chooses one good.
Your final assignment will be determined by a procedure that takes into account (i) the good you choose, (ii) the initial assignments, and (iii) the choices of the other participants. The procedure takes place over multiple stages, as described below.

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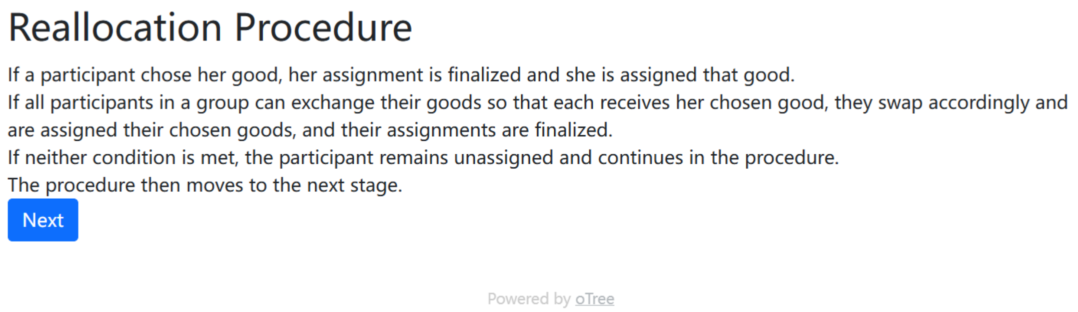
Figure C.19: Gradual TTC: Instruction 3



Reallocation Procedure

If a participant chose her good, her assignment is finalized and she is assigned that good.
If all participants in a group can exchange their goods so that each receives her chosen good, they swap accordingly and are assigned their chosen goods, and their assignments are finalized.
If neither condition is met, the participant remains unassigned and continues in the procedure.
The procedure then moves to the next stage.

Next

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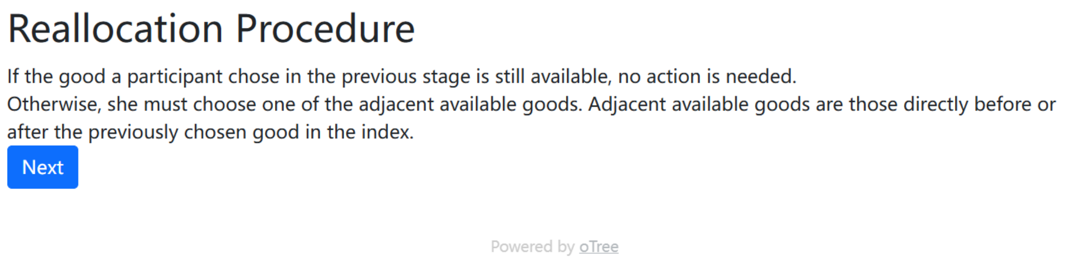
Figure C.20: Gradual TTC: Instruction 4



Reallocation Procedure

If the good a participant chose in the previous stage is still available, no action is needed.
Otherwise, she must choose one of the adjacent available goods. Adjacent available goods are those directly before or after the previously chosen good in the index.

Next

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Figure C.21: Gradual TTC: Instruction 5

Reallocation Procedure

If a participant chose her good, her assignment is finalized and she is assigned that good.
If all participants in a group can exchange their goods so that each receives her chosen good, they swap accordingly and are assigned their chosen goods, and their assignments are finalized.
If neither condition is met, the participant remains unassigned and continues in the procedure.
The procedure then moves to the next stage and repeats the same process until every participant is assigned a good.

Next

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Figure C.22: Gradual TTC: Instruction 6

Example 1

Consider an example with three participants. In the actual experiment, there are five participants.
The initial assignments are: Participant 1 with Good 1, Participant 2 with Good 2, and Participant 3 with Good 3. Suppose Participant 1 chooses Good 2, Participant 2 chooses Good 2, and Participant 3 chooses Good 1.
Since Participant 2 chose her own good, her assignment is finalized and she is assigned Good 2. Because Good 2 is no longer available, Participant 1 must choose again. The remaining goods are Good 1 and Good 3, both adjacent to Good 2. Therefore, Participant 1 may choose either Good 1 or Good 3. Suppose she chooses Good 3.
Since Participants 1 and 3 chose each other's good, they swap. Participant 1 is assigned Good 3 and Participant 3 is assigned Good 1, and their assignments are finalized.
As all participants are now assigned a good, the procedure ends.

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Figure C.23: Gradual TTC: Instruction 7

Example 2

Consider another example with three participants.
The initial assignments are: Participant 1 with Good 1, Participant 2 with Good 2, and Participant 3 with Good 3. Suppose Participant 1 chooses Good 3, Participant 2 chooses Good 1, and Participant 3 chooses Good 1.
Since Participants 1 and 3 chose each other's good, they swap. Participant 1 is assigned Good 3 and Participant 3 is assigned Good 1, and their assignment is finalized.
Because only Participant 2 and Good 2 remain, Participant 2 is assigned Good 2, and her assignment is finalized.
As all participants are now assigned a good, the procedure ends.

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Figure C.24: Gradual TTC: Instruction 8

C.2.2 Quiz

Quiz 1

Suppose the following table shows the points a participant receives from each good:

Good	Points
1	7
2	9
3	5
4	3
5	1

How many points does the participant receive if she is assigned Good 3?

○ 1 ○ 3 ○ 5 ○ 7 ○ 9

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Figure C.25: Gradual TTC: Question 1

Quiz 2

There are 5 goods, and a participant initially owns Good 2.

(1) The participant asked for Good 3 in stage 1 but did not receive it. The goods that remain available in stage 2 are Goods 1, 2, 4, and 5. Will the participant need to ask for a good in stage 2? If so, which good(s) can the participant ask for in stage 2?

☐ No need to ask ☐ Good 1 ☐ Good 2 ☐ Good 3 ☐ Good 4 ☐ Good 5

(2) The participant asked for Good 3 in stage 1 but did not receive it. The goods that remain available in stage 2 are Goods 2, 3, and 5. Will the participant need to ask for a good in stage 2? If so, which good(s) can the participant ask for in stage 2?

☐ No need to ask ☐ Good 1 ☐ Good 2 ☐ Good 3 ☐ Good 4 ☐ Good 5

(3) The participant asked for Good 3 in stage 1 and received it. The goods that remain available in stage 2 are Goods 1, 4, and 5. Will the participant need to ask for a good in stage 2? If so, which good(s) can the participant ask for in stage 2?

☐ No need to ask ☐ Good 1 ☐ Good 2 ☐ Good 3 ☐ Good 4 ☐ Good 5

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Figure C.26: Gradual TTC: Question 2

Quiz 3

There are 5 goods and 5 participants, and each of the 5 participants initially owns one of the 5 goods: Participant 1 owns Good 1, Participant 2 owns Good 2, Participant 3 owns Good 3, Participant 4 owns Good 4, and Participant 5 owns Good 5.

In stage 1, 5 participants asked for the following goods, respectively:
Participant 1: Good 2 Participant 2: Good 2 Participant 3: Good 4
Participant 4: Good 3 Participant 5: Good 3

(1) Is Participant 2's assignment **finalized** in stage 1? If so, which good is permanently assigned?

○ Not finalized ○ Good 1 ○ Good 2 ○ Good 3 ○ Good 4 ○ Good 5

(2) Is Participant 1's assignment **finalized** in stage 1? If so, which good is permanently assigned?

○ Not finalized ○ Good 1 ○ Good 2 ○ Good 3 ○ Good 4 ○ Good 5

(3) Which group of participants **exchange** their goods in stage 1?

○ Participants 1 and 2 ○ Participants 2, 3, and 5 ○ Participants 3 and 4 ○ None of them

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Figure C.27: Gradual TTC: Question 3

C.2.3 Main screens

Round1

· There are 5 goods.
· The points you receive from each good are shown in the table on the right. The good you initially own is highlighted in red.
· Please select a good, using the box below. You will only be shown goods you can choose from.

Good	Points
1	1
2	3
3	5
4	7
5	9

The good you are asking for -- Submit

Figure C.28: Gradual TTC: Main decision screen 1

Round1

· The good you currently own is highlighted in red.
· Goods that are no longer available are grayed out.
· Please select a good, using the box below. You will only be shown goods you can choose from.

Good	Points
1	1
2	3
3	5
4	7
5	9

The good you are asking for -- Submit

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Figure C.29: Gradual TTC: Main decision screen 2. Shown when necessary

Round1 result

· You are assigned Good 4.
· The table showing the points you receive from each good is displayed in the left panel, while the list of goods you have requested is displayed in the right panel as a summary.

Good	Points
1	1
2	3
3	5
4	7
5	9

Stage 1	3
Stage 2	4

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Figure C.30: Gradual TTC: End of a round feedbaclk

C.3 Static Crawler

C.3.1 Instructions

Overview

In this experiment, each participant initially owns one good. These goods will be reallocated among participants through a specific procedure. Your earnings from the experiment will be paid to you in cash at the end. The entire experiment will take about 90 minutes.

Next

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Figure C.31: Static Crawler: Instruction 1

Initial Setup

You will be randomly assigned to a group of five participants. At the beginning, you will see a screen showing the good you initially own and the points you receive from each good. The higher the points, the more desirable the good is for you.
The screen below is provided as an example.
In this example, the participant initially owns Good 3.
She receives 3 points if assigned Good 1, 7 points if assigned Good 2, 9 points if assigned Good 3, 5 points if assigned Good 4, and 1 point if assigned Good 5.

- There are 5 goods.
- The points you receive from each good are shown in the table on the right. The good you initially own is highlighted in red.
- Please submit your ranking, using the panel below.

Good	Points
1	3
2	7
3	9
4	5
5	1

Rank 1 --
Rank 2 --
Rank 3 --
Rank 4 --
Rank 5 --

Submit

This is an example screenshot.

Next

Figure C.32: Static Crawler: Instruction 2

Reallocation Procedure

You will submit a ranking of the goods from most to least preferred. Each good must appear exactly once in your ranking.
Goods are indexed from 1 to 5. If you choose Good k as your first choice, your second choice must be one of its adjacent goods: either Good $k-1$ or Good $k+1$. Note that Goods 1 and 5 each have only one adjacent good: Good 2 is adjacent to Good 1, and Good 4 is adjacent to Good 5.
Your third choice must follow the same principle. For example, if your second choice is Good $k+1$, then your third choice must be either Good $k+2$ (continuing in the same direction) or Good $k-1$ (switching direction and moving one step away from your first choice). Similarly, if your second choice is Good $k-1$, your third choice must be either Good $k-2$ or Good $k+1$.
This rule continues until you have ranked all goods: at each step you can either move one step further in the same direction or switch direction and move one step away from your first choice, without skipping any goods.

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Figure C.33: Static Crawler: Instruction 3

Reallocation Procedure

For example, if you select Good 1 as your top choice, your second choice must be Good 2, your third Good 3, and your fourth Good 4. If you select Good 2 as your top choice, your second choice must be either Good 1 or Good 3. If you then select Good 1 as your second choice, your third must be Good 3. On the other hand, if you select Good 3 as your second choice, your third may be either Good 1 or Good 4.

Your final assignment will be determined by a procedure that takes into account (i) the ranking you submit, (ii) the initial assignments, and (iii) the rankings submitted by the other participants. The procedure takes place over multiple steps, as described below.

Next

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Figure C.34: Static Crawler: Instruction 4

Reallocation Procedure

At the first step, all goods are available to all participants.

At each step:

- If a participant ranks her good highest among the remaining goods, her assignment is finalized and she is assigned that good.
- If two participants whose goods are adjacent among the remaining ones each rank the other's good higher than her own, they swap goods. After a swap, their assignments are not yet finalized; they remain in the procedure while holding their new goods.

The procedure continues until every participant's assignment is finalized.

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Figure C.35: Static Crawler: Instruction 5

Example 1

Consider an example with three participants. In the actual experiment, there are five participants. The initial assignments are: Participant 1 with Good 1, Participant 2 with Good 2, and Participant 3 with Good 3. Suppose the participants submit the rankings shown on the right.
Since Participant 2 ranks Good 2 highest, her assignment is finalized and she is assigned Good 2. Because Good 2 is no longer available, Goods 1 and 3 are adjacent. Since Participants 1 and 3 each rank the other's good higher than her own, they swap. As a result, Participant 1 now holds Good 3 and Participant 3 now holds Good 1.
Now, since both Participants 1 and 3 rank their current goods highest among the remaining goods, their assignments are finalized: Participant 1 is assigned Good 3 and Participant 3 is assigned Good 1.
As all participants are now assigned a good, the procedure ends.

Participant 1's Ranking
Good 2
Good 3
Good 1

Participant 2's Ranking
Good 2
Good 1
Good 3

Participant 3's Ranking
Good 1
Good 2
Good 3

Next

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Figure C.36: Static Crawler: Instruction 6

Example 2

Consider another example with three participants.
The initial assignments are: Participant 1 with Good 1, Participant 2 with Good 2, and Participant 3 with Good 3. Suppose the participants submit the rankings shown on the right.
Since Participants 1 and 2 each rank the other's good higher than their own, they swap. As a result, Participant 1 now holds Good 2 and Participant 2 now holds Good 1. Since Participant 2 ranks Good 1 highest, her assignment is finalized and she is assigned Good 1.
Since Participants 1 and 3 each rank the other's good higher than their own, they swap. As a result, Participant 1 now holds Good 3 and Participant 3 now holds Good 2.
Now, since both Participants 1 and 3 rank their current goods highest among the remaining goods, their assignments are finalized: Participant 1 is assigned Good 3 and Participant 3 is assigned Good 2.
As all participants are now assigned a good, the procedure ends.

Participant 1's Ranking
Good 3
Good 2
Good 1

Participant 2's Ranking
Good 1
Good 2
Good 3

Participant 3's Ranking
Good 1
Good 2
Good 3

Next

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Figure C.37: Static Crawler: Instruction 7

Rounds and Payment

You will participate in this type of allocation problem for 20 rounds. In each round, you will have a different initial good and different points for each good, and you will play with different participants.
You will receive a show-up fee of ¥750. At the end of the experiment, one round will be randomly selected, and your total earnings will be based on your earnings from that round. The conversion rate is 1 point = ¥250.

Next

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Figure C.38: Static Crawler: Instruction 8

C.3.2 Quiz

Quiz 1

Suppose the following table shows the points a participant receives from each good:

Good	Points
1	7
2	9
3	5
4	3
5	1

How many points does the participant receive if she is assigned Good 3?

○ 1 ○ 3 ○ 5 ○ 7 ○ 9

Next

▶ Instruction

Figure C.39: Static Crawler: Question 1

Quiz 2

Suppose there are 5 goods, and a participant initially owns Good 2.

(1) The participant selects Good 3 as their first choice. Which good(s) can the participant report as their second choice?

☐ Good 1 ☐ Good 2 ☐ Good 3 ☐ Good 4 ☐ Good 5

(2) The participant selects Good 4 as their first choice and Good 5 as their second choice. Which good(s) can the participant report as their third choice?

☐ Good 1 ☐ Good 2 ☐ Good 3 ☐ Good 4 ☐ Good 5

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Figure C.40: Static Crawler: Question 2

Quiz 3

There are 5 goods, and the rankings below were submitted by 5 participants. For each participant, their initial good is highlighted in red in their respective table.

Participant 1	Participant 2	Participant 3	Participant 4	Participant 5
Good 2	Good 2	Good 4	Good 1	Good 3
Good 3	Good 3	Good 5	Good 2	Good 2
Good 1	Good 4	Good 3	Good 3	Good 4
Good 4	Good 5	Good 2	Good 4	Good 1
Good 5	Good 1	Good 1	Good 5	Good 5

(1) Which good is Participant 2 assigned?

○ Good 1 ○ Good 2 ○ Good 3 ○ Good 4 ○ Good 5

(2) Which pair(s) of participants will swap their initial goods?

○ Participants 1 and 2 ○ Participants 1 and 3 ○ Participants 3 and 4 ○ None of them

(3) Which good is Participant 5 assigned?

○ Good 1 ○ Good 2 ○ Good 3 ○ Good 4 ○ Good 5

Next

Figure C.41: Static Crawler: Question 3

C.3.3 Main screens

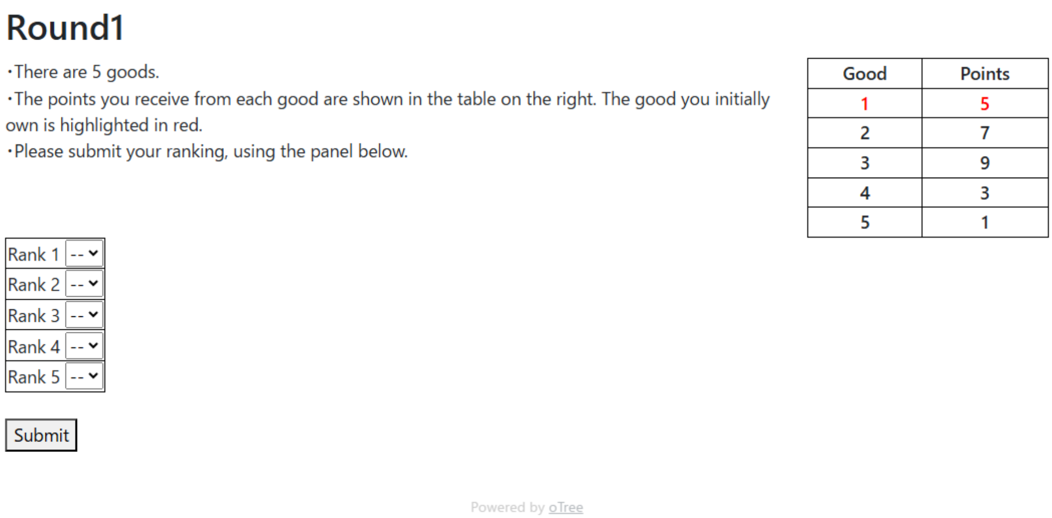

Figure C.42: Static Crawler: Main decision screen

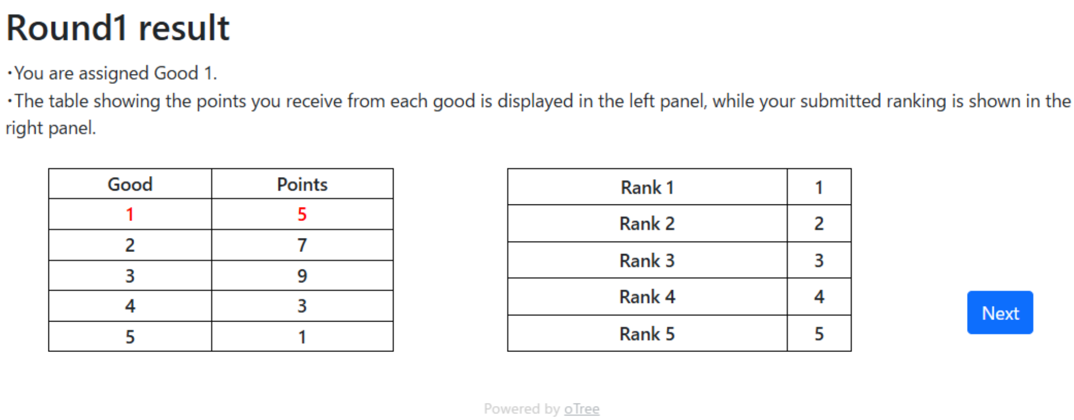

Figure C.43: Static Crawler: End of a round feedback

C.4 Static TTC

C.4.1 Instructions

Overview

In this experiment, each participant initially owns one good. These goods will be reallocated among participants through a specific procedure. Your earnings from the experiment will be paid to you in cash at the end. The entire experiment will take about 90 minutes.

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Figure C.44: Static TTC: Instruction 1

Initial Setup

You will be randomly assigned to a group of five participants. At the beginning, you will see a screen showing the good you initially own and the points you receive from each good. The higher the points, the more desirable the good is for you.
The screen below is provided as an example.
In this example, the participant initially owns Good 3.
She receives 3 points if assigned Good 1, 7 points if assigned Good 2, 9 points if assigned Good 3, 5 points if assigned Good 4, and 1 point if assigned Good 5.

- There are 5 goods.
- The points you receive from each good are shown in the table on the right. The good you initially own is highlighted in red.
- Please submit your ranking, using the panel below.

Good	Points
1	3
2	7
3	9
4	5
5	1

Rank 1	--
Rank 2	--
Rank 3	--
Rank 4	--
Rank 5	--

Submit

This is an example screenshot.

Next

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Figure C.45: Static TTC: Instruction 2

Reallocation Procedure

You will submit a ranking of the goods from most to least preferred. Each good must appear exactly once in your ranking.
Goods are indexed from 1 to 5. If you choose Good k as your first choice, your second choice must be one of its adjacent goods: either Good k − 1 or Good k+1. Note that Goods 1 and 5 each have only one adjacent good: Good 2 is adjacent to Good 1, and Good 4 is adjacent to Good 5.
Your third choice must follow the same principle. For example, if your second choice is Good k+1, then your third choice must be either Good k+2 (continuing in the same direction) or Good k − 1 (switching direction and moving one step away from your first choice). Similarly, if your second choice is Good k − 1, your third choice must be either Good k − 2 or Good k+1.
This rule continues until you have ranked all goods: at each step you can either move one step further in the same direction or switch direction and move one step away from your first choice, without skipping any goods.

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Figure C.46: Static TTC: Instruction 3

Reallocation Procedure

For example, if you select Good 1 as your top choice, your second choice must be Good 2, your third Good 3, and your fourth Good 4.
If you select Good 2 as your top choice, your second choice must be either Good 1 or Good 3. If you then select Good 1 as your second choice, your third must be Good 3. On the other hand, if you select Good 3 as your second choice, your third may be either Good 1 or Good 4.
Your final assignment will be determined by a procedure that takes into account (i) the ranking you submit, (ii) the initial assignments, and (iii) the rankings submitted by the other participants. The procedure takes place over multiple steps, as described below.

Next

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Figure C.47: Static TTC: Instruction 4

Reallocation Procedure

At the first step, all goods are available to all participants.
At each step:

- If a participant ranks her own good highest among the remaining goods, her assignment is finalized and she is assigned that good.
- If all participants in a group can exchange goods so that each receives her highest-ranked remaining good, they exchange accordingly and are assigned their goods, and their assignments are finalized.
- Otherwise, the participant remains unassigned and continues in the procedure.

The procedure continues until every participant is assigned a good.

Next

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Figure C.48: Static TTC: Instruction 5

Example 1

Consider an example with three participants. In the actual experiment, there are five participants. The initial assignments are: Participant 1 with Good 1, Participant 2 with Good 2, and Participant 3 with Good 3. Suppose the participants submit the rankings shown on the right.
Since Participant 2 ranks Good 2 highest, her assignment is finalized and she is assigned Good 2. Because Good 2 is no longer available, Participant 1 ranks Good 3 highest among the remaining goods, and Participant 3 ranks Good 1 highest among the remaining goods. They exchange, and their assignments are finalized: Participant 1 is assigned Good 3 and Participant 3 is assigned Good 1.
As all participants are now assigned a good, the procedure ends.

Participant 1's Ranking
Good 2
Good 3
Good 1

Participant 2's Ranking
Good 2
Good 1
Good 3

Participant 3's Ranking
Good 1
Good 2
Good 3

Next

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Figure C.49: Static TTC: Instruction 6

Example 2

Consider another example with three participants.
The initial assignments are: Participant 1 with Good 1, Participant 2 with Good 2, and Participant 3 with Good 3. Suppose the participants submit the rankings shown on the right.
Participant 1 ranks Good 3 highest, and Participant 3 ranks Good 1 highest. They exchange, and their assignments are finalized: Participant 1 is assigned Good 3 and Participant 3 is assigned Good 1.
Because only Participant 2 and Good 2 remain, Participant 2 is assigned Good 2.
As all participants are now assigned a good, the procedure ends.

Participant 1's Ranking
Good 3
Good 2
Good 1

Participant 2's Ranking
Good 1
Good 2
Good 3

Participant 3's Ranking
Good 1
Good 2
Good 3

Next

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Figure C.50: Static TTC: Instruction 7

Rounds and Payment

You will participate in this type of allocation problem for 20 rounds. In each round, you will have a different initial good and different points for each good, and you will play with different participants.
You will receive a show-up fee of ¥750. At the end of the experiment, one round will be randomly selected, and your total earnings will be based on your earnings from that round. The conversion rate is 1 point = ¥250.

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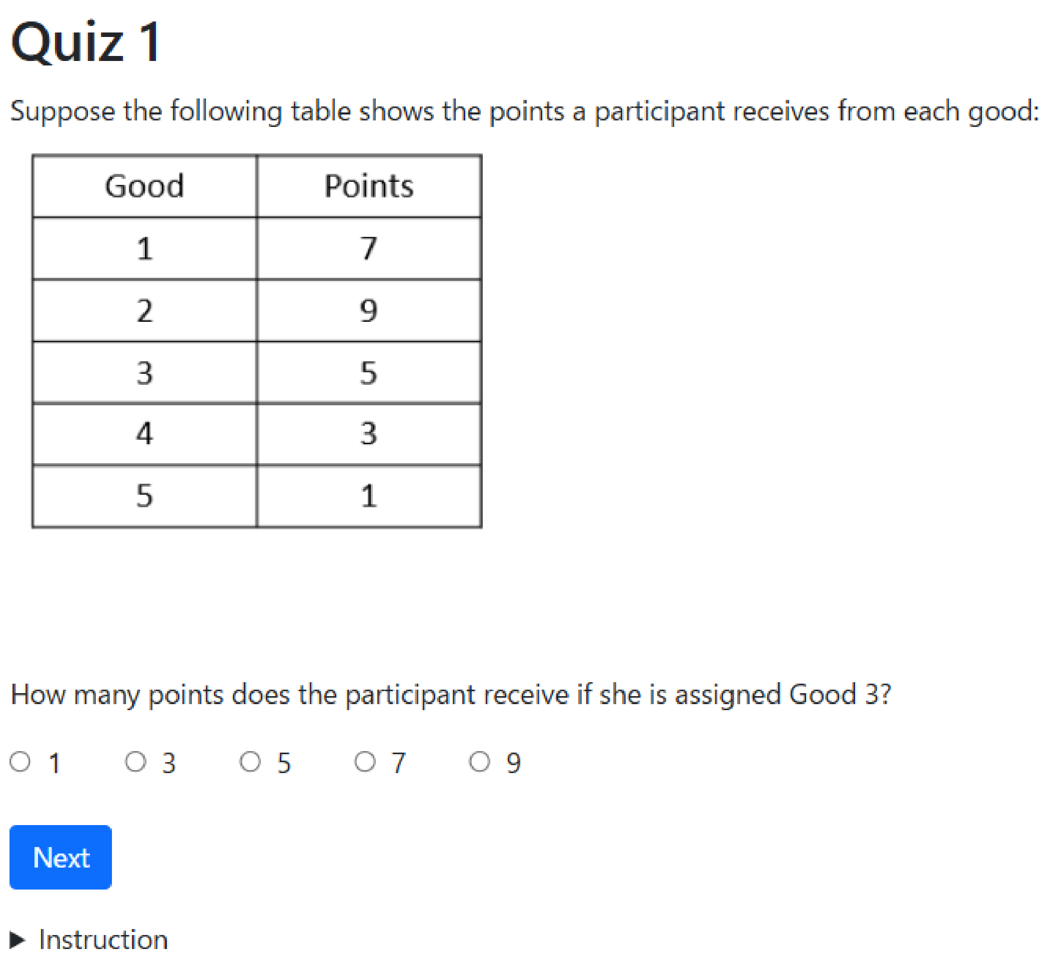
Figure C.51: Static TTC: Instruction 8

C.4.2 Quiz



Quiz 1

Suppose the following table shows the points a participant receives from each good:

Good	Points
1	7
2	9
3	5
4	3
5	1

How many points does the participant receive if she is assigned Good 3?

○ 1 ○ 3 ○ 5 ○ 7 ○ 9

Next

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Figure C.52: Static TTC: Question 1

Quiz 2

Suppose there are 5 goods, and a participant initially owns Good 2.

(1) The participant selects Good 3 as their first choice. Which good(s) can the participant report as their second choice?

☐ Good 1 ☐ Good 2 ☐ Good 3 ☐ Good 4 ☐ Good 5

(2) The participant selects Good 4 as their first choice and Good 5 as their second choice. Which good(s) can the participant report as their third choice?

☐ Good 1 ☐ Good 2 ☐ Good 3 ☐ Good 4 ☐ Good 5

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Figure C.53: Static TTC: Question 2

Quiz 3

There are 5 goods, and the rankings below were submitted by 5 participants. For each participant, their initial good is highlighted in red in their respective table.

Participant 1	Participant 2	Participant 3	Participant 4	Participant 5
Good 2	Good 2	Good 4	Good 1	Good 3
Good 3	Good 3	Good 5	Good 2	Good 2
Good 1	Good 4	Good 3	Good 3	Good 4
Good 4	Good 5	Good 2	Good 4	Good 1
Good 5	Good 1	Good 1	Good 5	Good 5

(1) Which good is Participant 2 assigned?

○ Good 1 ○ Good 2 ○ Good 3 ○ Good 4 ○ Good 5

(2) Which group(s) of participants will exchange their initial goods?

○ Participants 1 and 2 ○ Participants 1, 3, and 4 ○ Participants 1, 4, and 5 ○ None of them

(3) Which good is Participant 5 assigned?

○ Good 1 ○ Good 2 ○ Good 3 ○ Good 4 ○ Good 5

Next

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Figure C.54: Static TTC: Question 3

C.4.3 Main screens

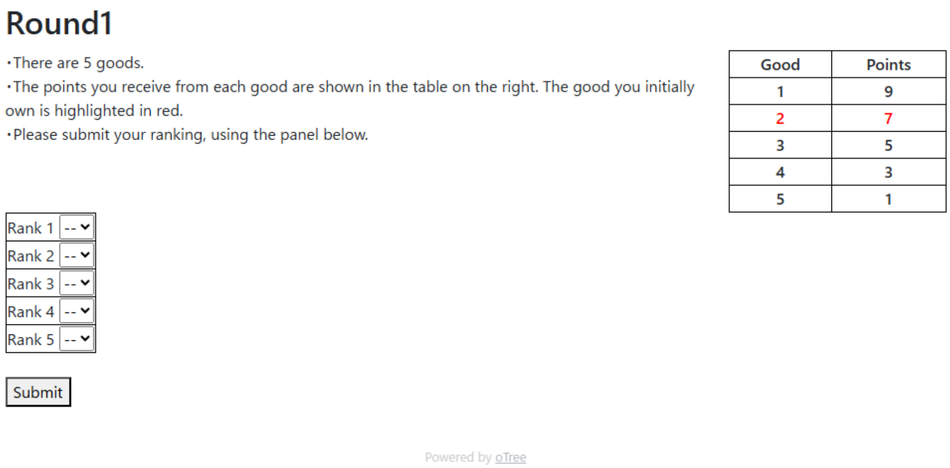

Figure C.55: Static TTC: Main decision screen

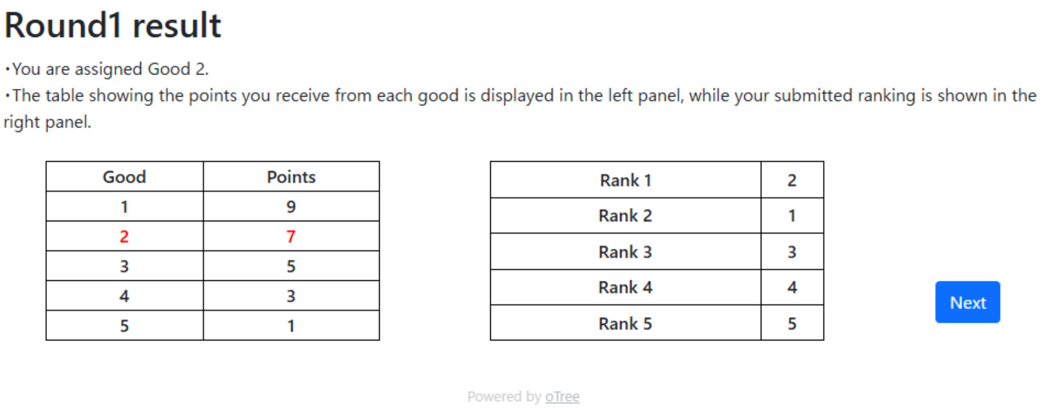

Figure C.56: Static TTC: End of a round feedback

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